Algorithms for NLP

Parsing / Classification I

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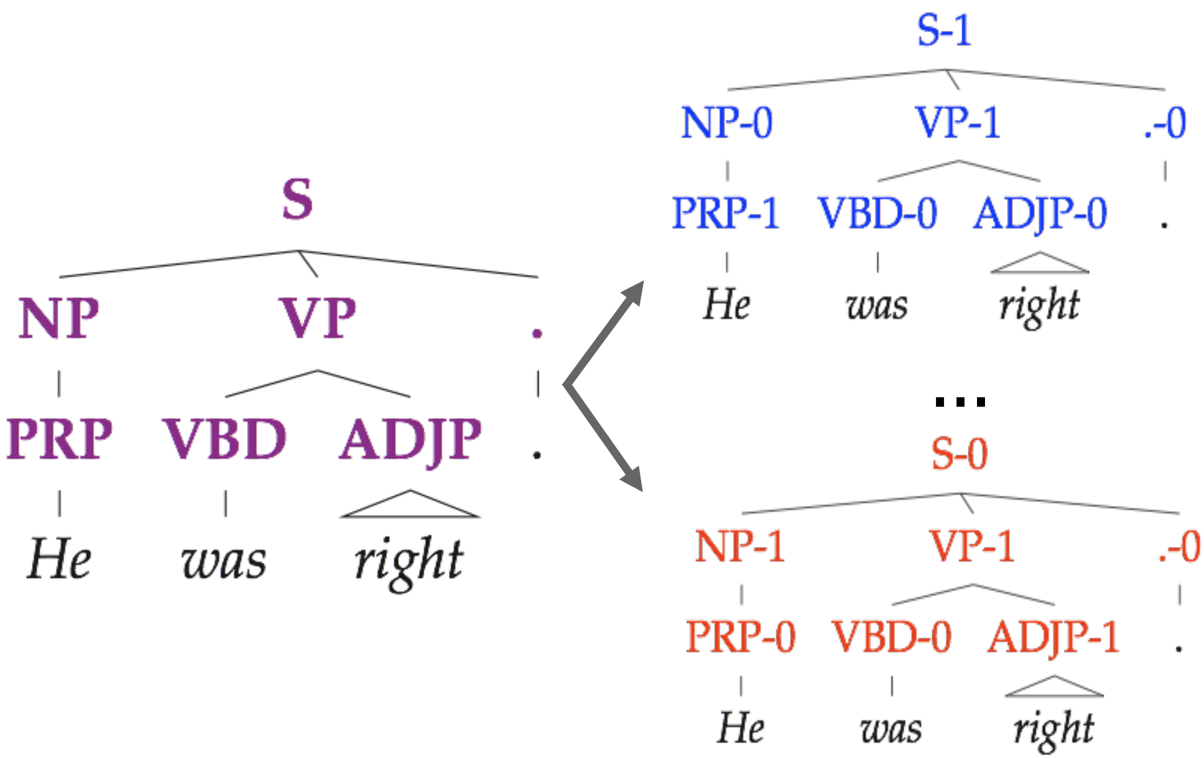
Latent Variable Grammars



Grammar G

$S_0 \to NP_0\ VP_0$?
$S_0 \to NP_1\ VP_0$?
$S_0 \to NP_0\ VP_1$?
$S_0 \to NP_1\ VP_1$?
$S_1 \to NP_0\ VP_0$?
...
$S_1 \to NP_1\ VP_1$?
...
$NP_0 \to PRP_0$?
$NP_0 \to PRP_1$?
...

Lexicon

$PRP_0 \to$ She ?
$PRP_1 \to$ She ?
...
$VBD_0 \to$ was ?
$VBD_1 \to$ was ?
$VBD_2 \to$ was ?
...

Parse Tree T
Sentence w

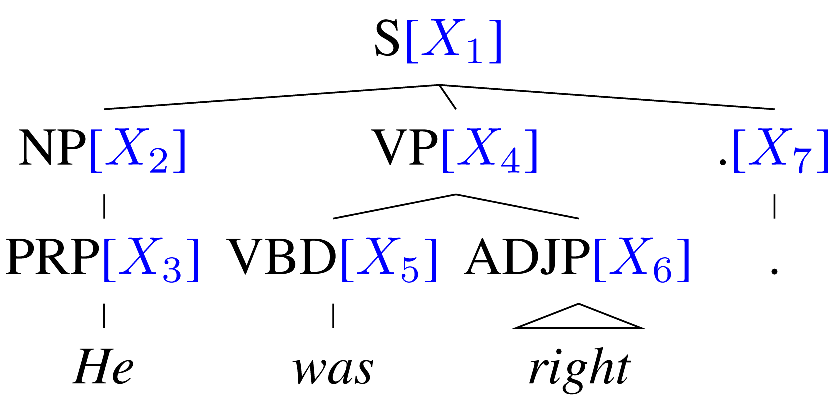
Derivations $t : T$

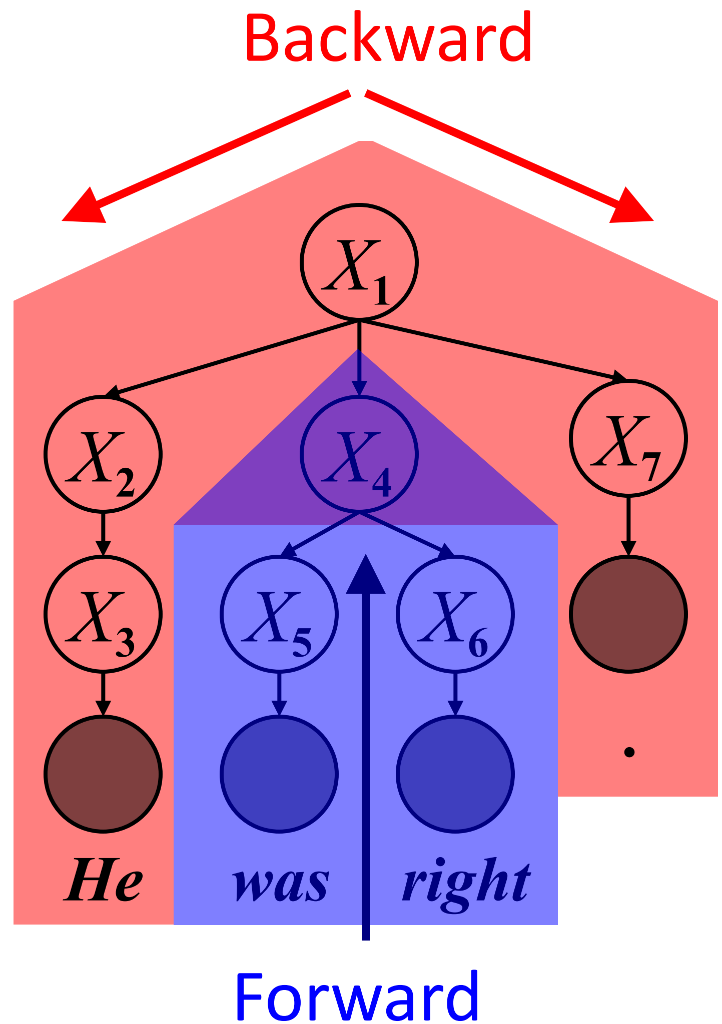
Parameters θ

Learning Latent Annotations

EM algorithm:

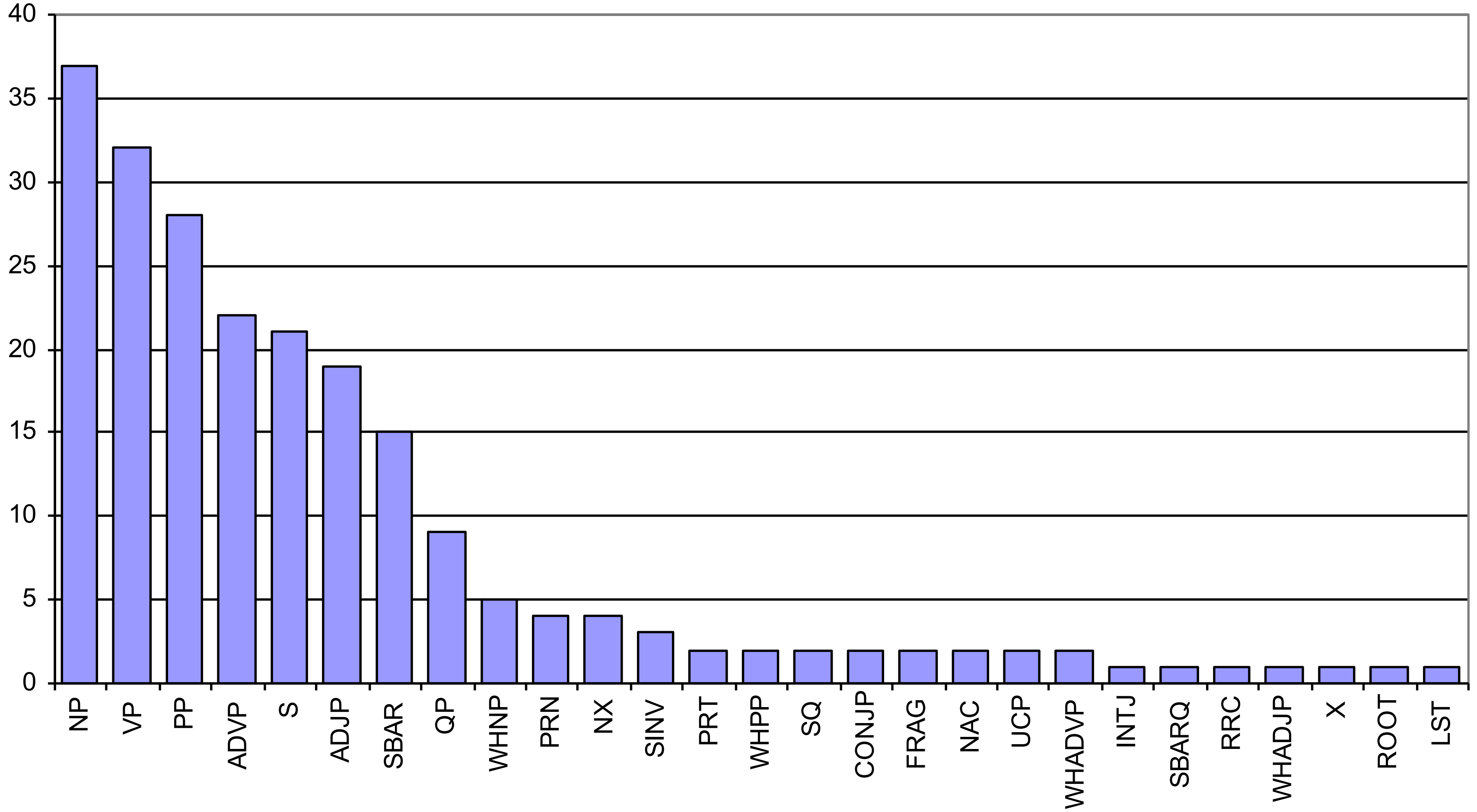
- Brackets are known
- Base categories are known
- Only induce subcategories



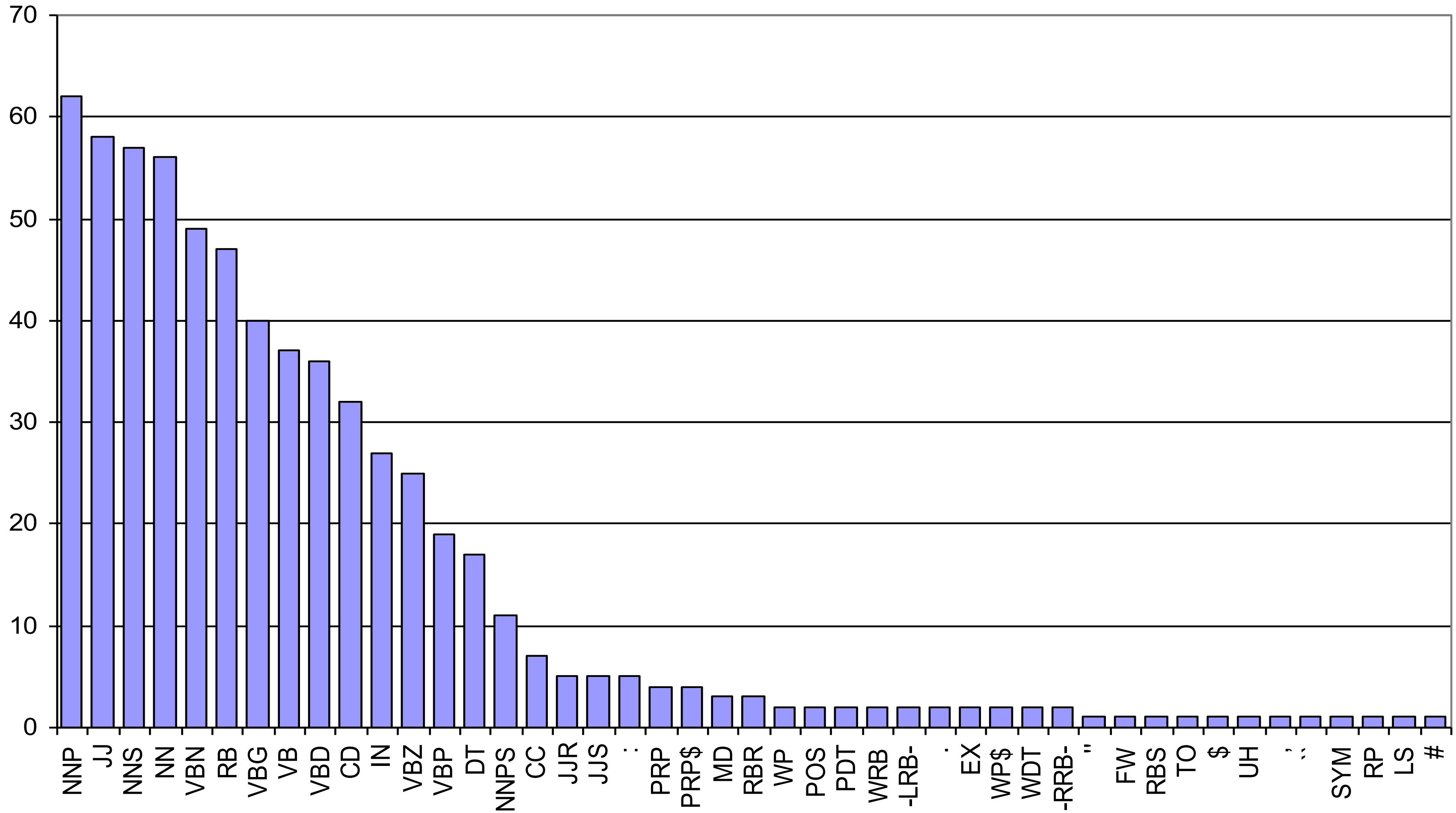


Just like Forward-Backward for HMMs.

Number of Phrasal Subcategories



Number of Lexical Subcategories



Learned Splits

- Proper Nouns (NNP):

NNP-14	Oct.	Nov.	Sept.
NNP-12	John	Robert	James
NNP-2	J.	E.	L.
NNP-1	Bush	Noriega	Peters
NNP-15	New	San	Wall
NNP-3	York	Francisco	Street

- Personal pronouns (PRP):

PRP-0	It	He	I
PRP-1	it	he	they
PRP-2	it	them	him

Learned Splits

- Relative adverbs (RBR):

RBR-0	further	lower	higher
RBR-1	more	less	More
RBR-2	earlier	Earlier	later

- Cardinal Numbers (CD):

CD-7	one	two	Three
CD-4	1989	1990	1988
CD-11	million	billion	trillion
CD-0	1	50	100
CD-3	1	30	31
CD-9	78	58	34

Final Results (Accuracy)

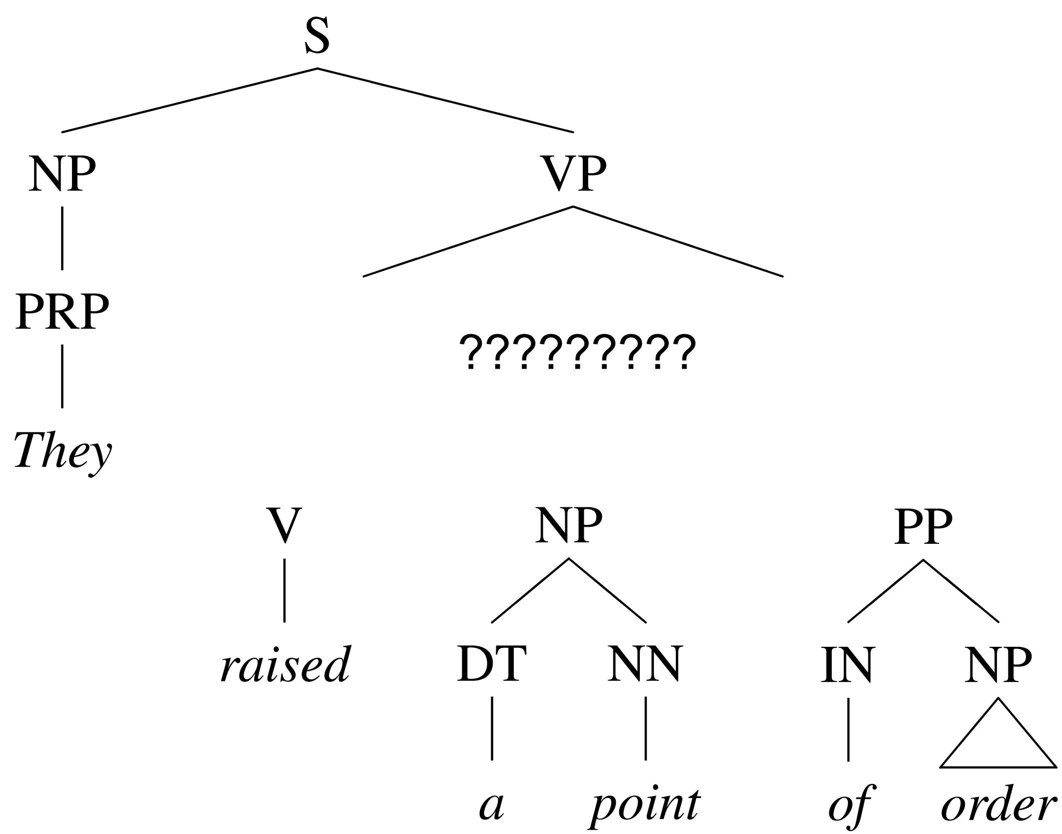
		≤ 40 words F1	all F1
ENG	Charniak&Johnson '05 (generative)	90.1	89.6
	Split / Merge	**90.6**	**90.1**
GER	Dubey '05	76.3	-
	Split / Merge	**80.8**	**80.1**
CHN	Chiang et al. '02	80.0	76.6
	Split / Merge	**86.3**	**83.4**

Still higher numbers from reranking / self-training methods

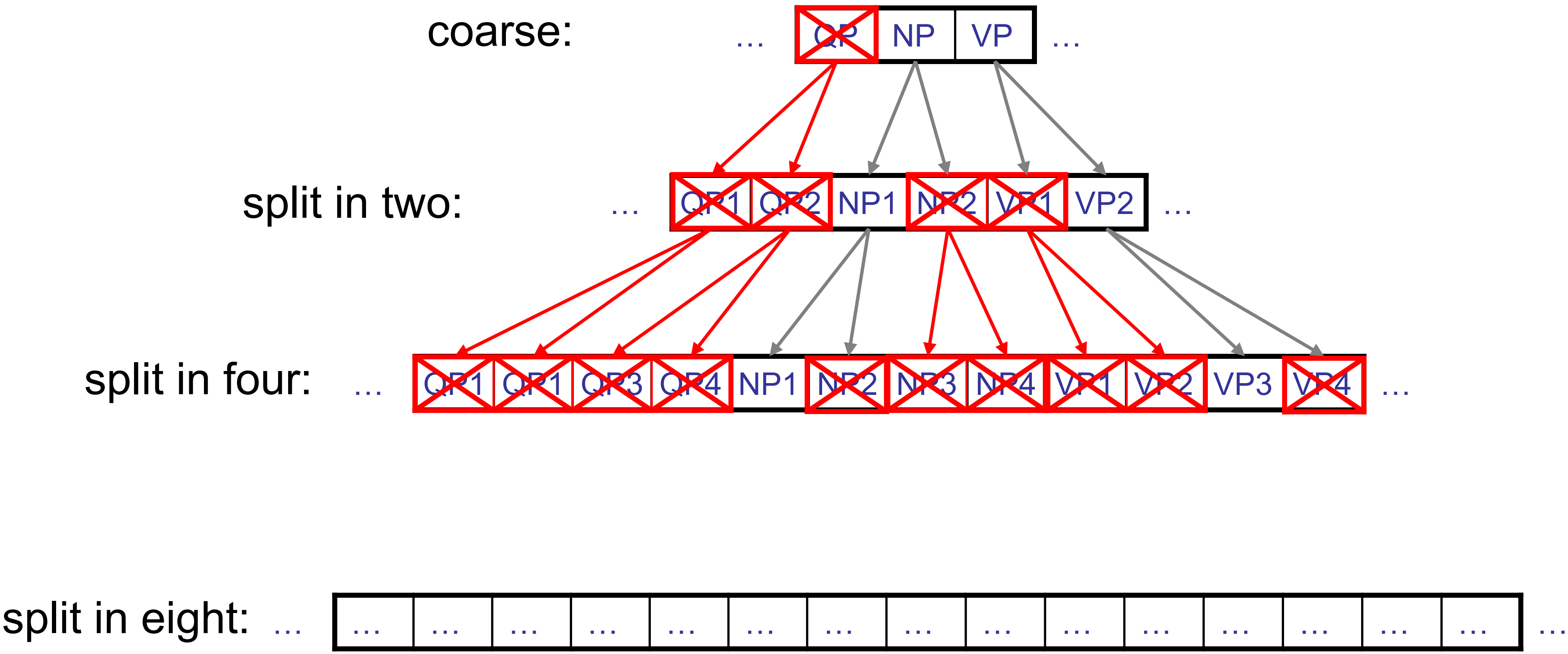
Efficient Parsing for Hierarchical Grammars

Coarse-to-Fine Inference

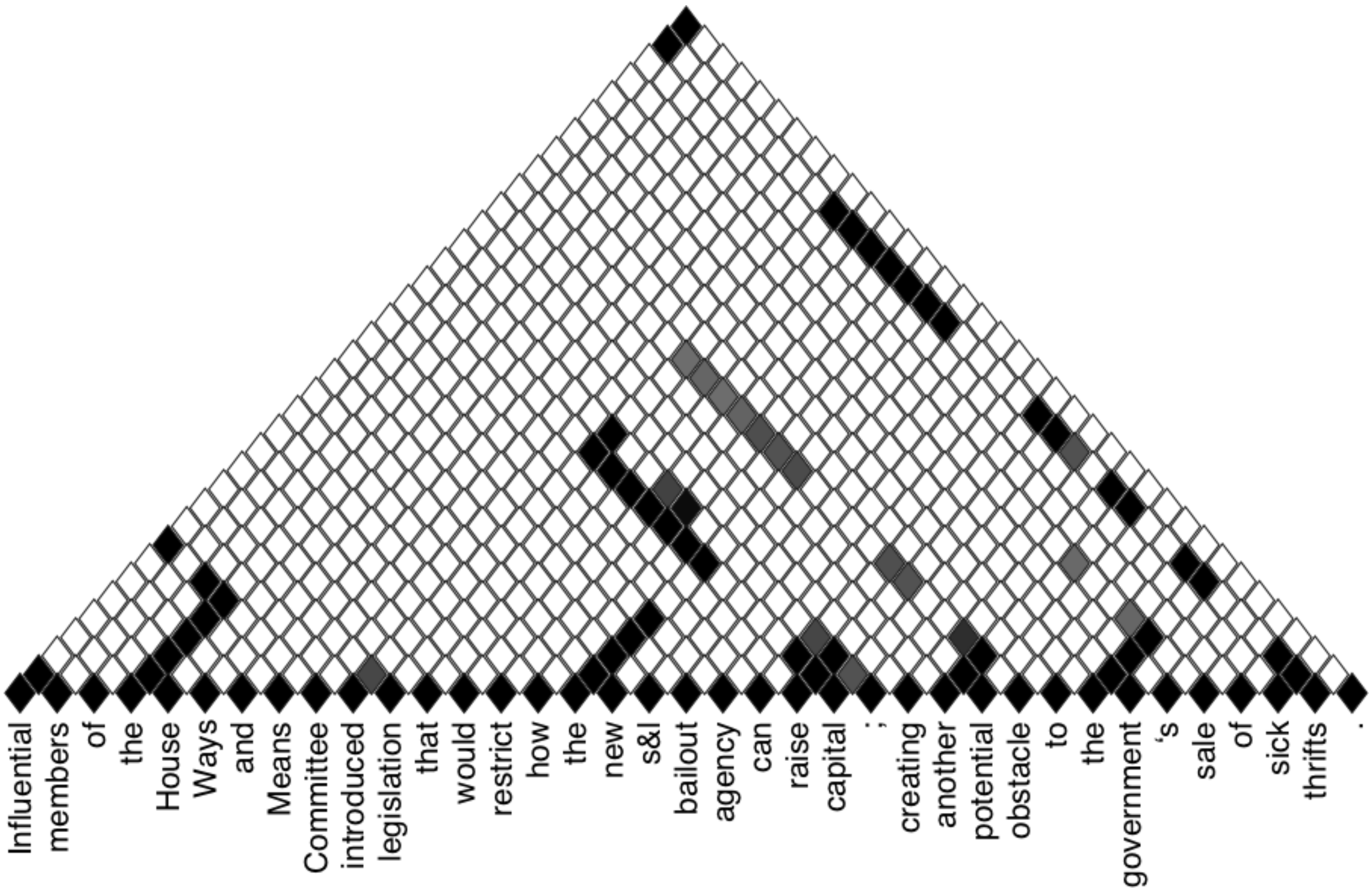
- Example: PP attachment



Hierarchical Pruning



Bracket Posteriors



1621 min

111 min

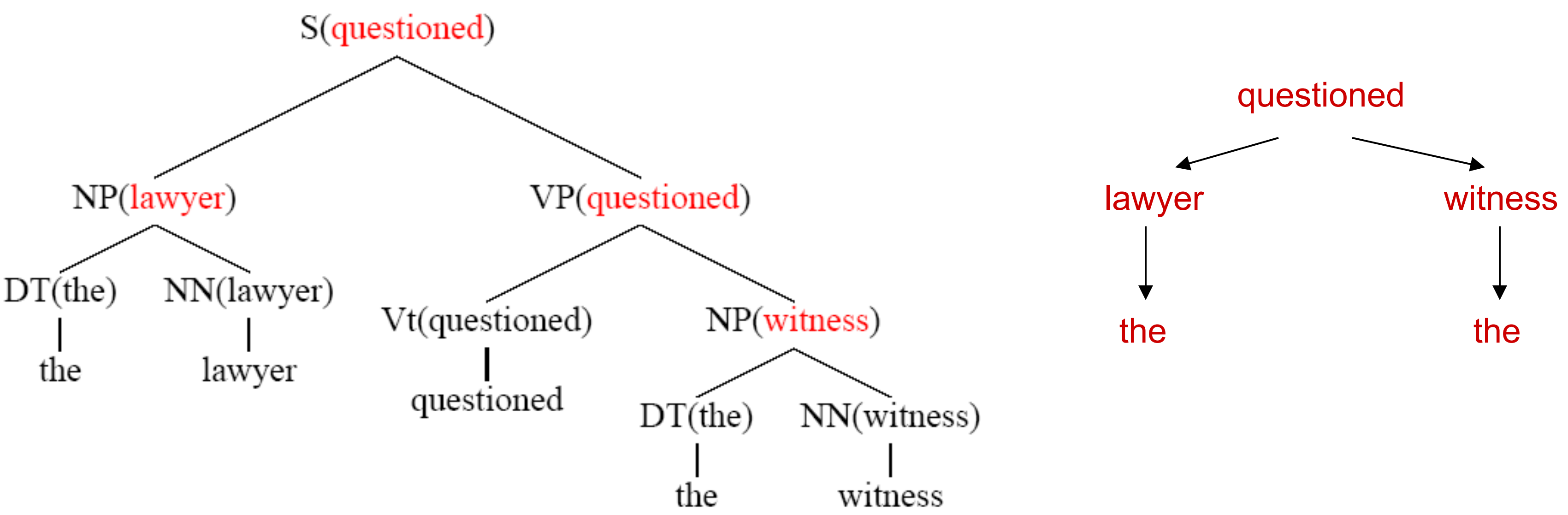
35 min

15 min
(no search error)

Other Syntactic Models

Dependency Parsing

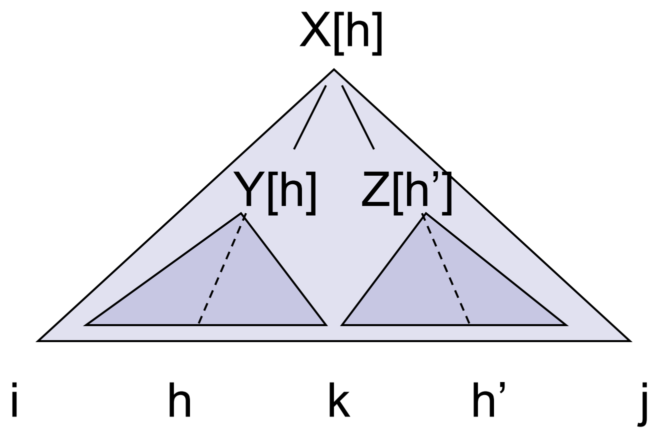
- Lexicalized parsers can be seen as producing *dependency trees*

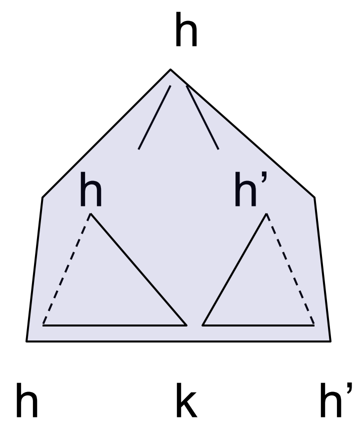


- Each local binary tree corresponds to an attachment in the dependency graph

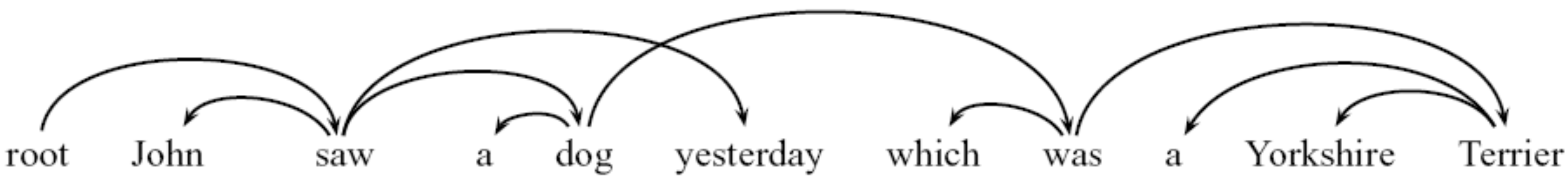
Dependency Parsing

- Pure dependency parsing is only cubic [Eisner 99]



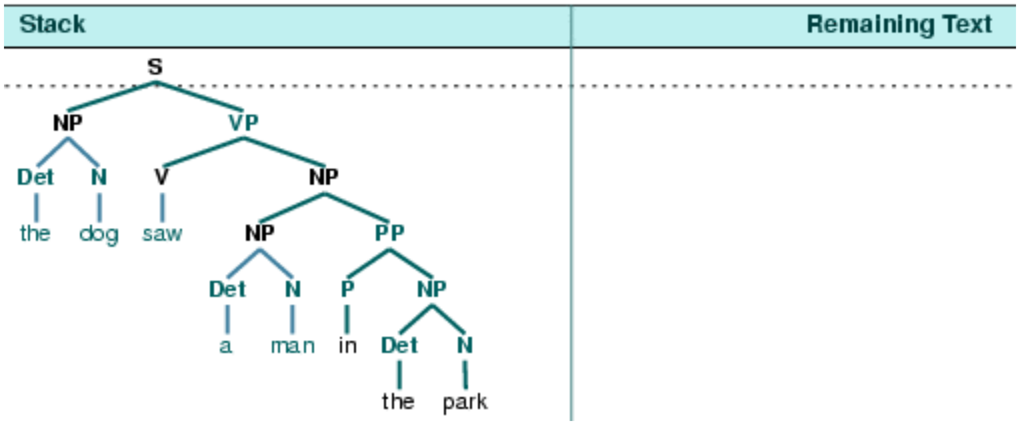


- Some work on *non-projective* dependencies
  - Common in, e.g. Czech parsing
  - Can do with MST algorithms [McDonald and Pereira 05]



Shift-Reduce Parsers

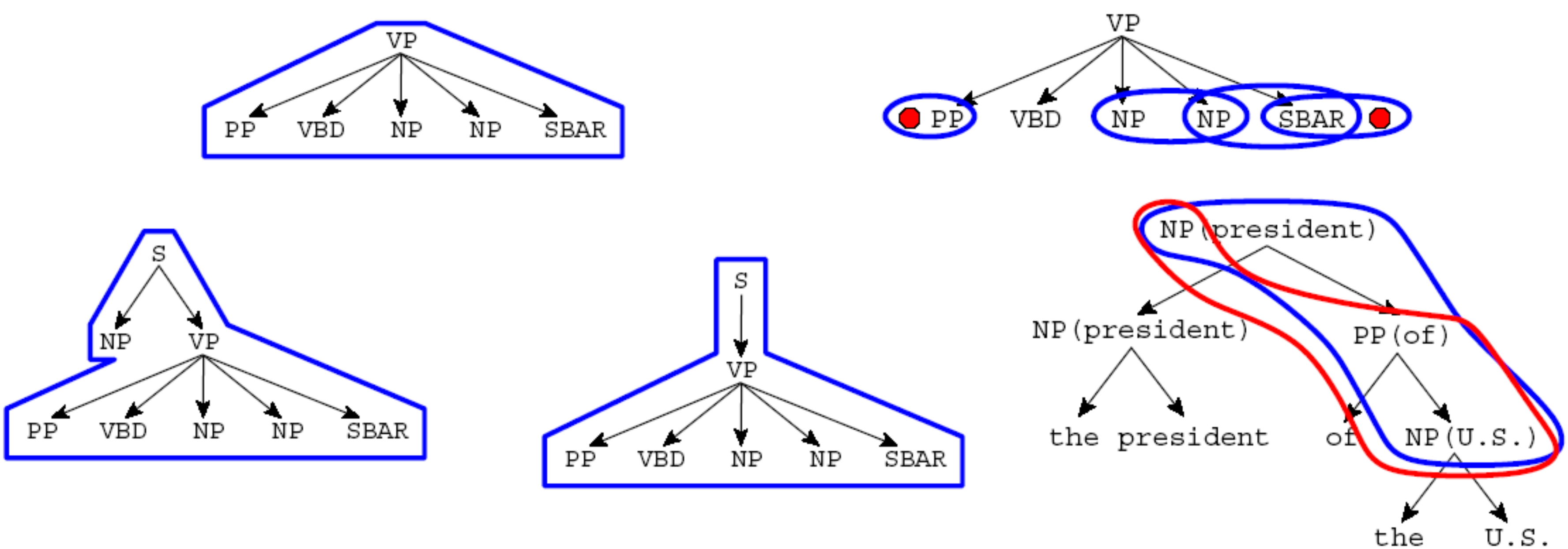
- Another way to derive a tree:



- Parsing
  - No useful dynamic programming search
  - Can still use beam search [Ratnaparkhi 97]

Parse Reranking

- Assume the number of parses is very small
- We can represent each parse T as a feature vector $\varphi(T)$
  - Typically, all local rules are features
  - Also non-local features, like how right-branching the overall tree is
  - [Charniak and Johnson 05] gives a rich set of features



Classification

Classification

- Automatically make a decision about inputs
  - Example: document $\rightarrow$ category
  - Example: image of digit $\rightarrow$ digit
  - Example: image of object $\rightarrow$ object type
  - Example: query + webpages $\rightarrow$ best match
  - Example: symptoms $\rightarrow$ diagnosis
  - …
- Three main ideas
  - Representation as feature vectors
  - Scoring by linear functions (or not, actually)
  - Learning by optimization

Some Definitions

INPUTS	$\mathbf{x}_i$	*close the ____*
CANDIDATE SET	$\mathcal{Y}(\mathbf{x})$	*{door, table, …}*
CANDIDATES	$\mathbf{y}$	*table*
TRUE OUTPUTS	$\mathbf{y}_i^*$	*door*
FEATURE VECTORS	$\mathbf{f}(\mathbf{x}, \mathbf{y})$	[0 0 1 0 0 0 1 0 0 0 0 0]

x_{-1}=“the” $\wedge$ y=“door”

x_{-1}=“the” $\wedge$ y=“table”

“close” in x $\wedge$ y=“door”

y occurs in x

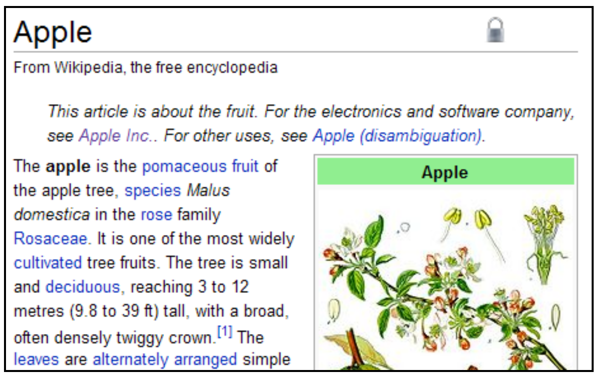
Features

Feature Vectors

- Example: web page ranking (not actually classification)

x_i = "Apple Computers"

$\mathbf{f}_i($



Apple

From Wikipedia, the free encyclopedia

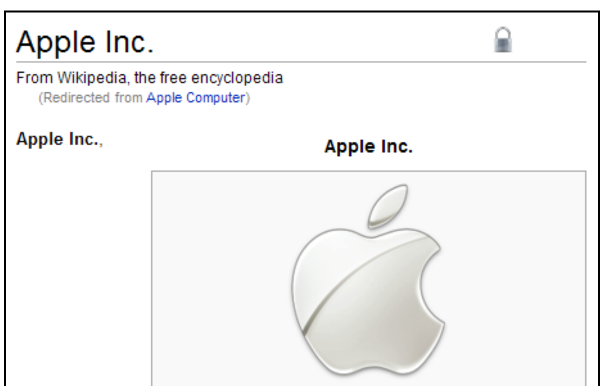
This article is about the fruit. For the electronics and software company, see Apple Inc.. For other uses, see Apple (disambiguation).

The **apple** is the pomaceous fruit of the apple tree, species *Malus domestica* in the rose family Rosaceae. It is one of the most widely cultivated tree fruits. The tree is small and deciduous, reaching 3 to 12 metres (9.8 to 39 ft) tall, with a broad, often densely twiggy crown.[1] The leaves are alternately arranged simple

Apple

$) = [0.3\ 5\ 0\ 0\ \ldots]$

$\mathbf{f}_i($



Apple Inc.

From Wikipedia, the free encyclopedia

(Redirected from Apple Computer)

Apple Inc.,

Apple Inc.

$) = [0.8\ 4\ 2\ 1\ \ldots]$

Block Feature Vectors

- Sometimes, we think of the input as having features, which are multiplied by outputs to form the candidates

$\mathbf{x}$ … *win the election* …

"$\mathbf{f}(\mathbf{x})$" $[1\ 0\ 1\ 0\ ]$

"*win*" "*election*"

… *win the election* …

$$\mathbf{f}(\mathit{SPORTS}) = [1\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]$$

… *win the election* …

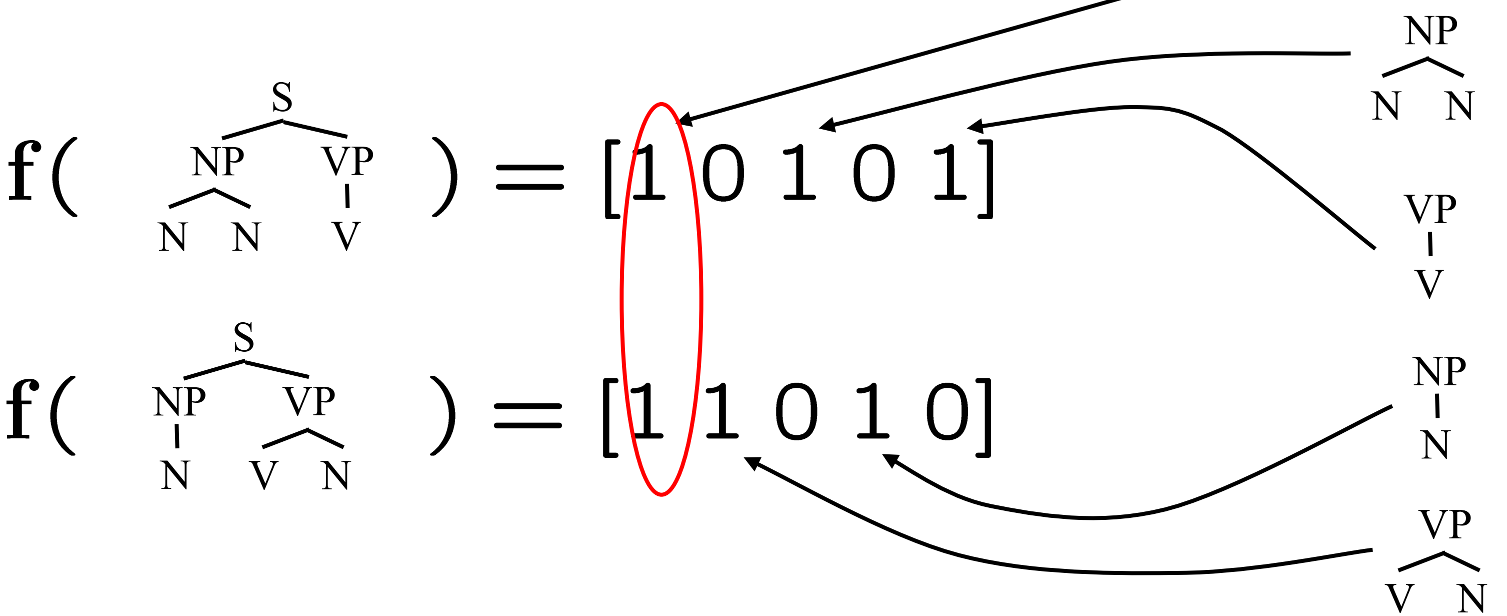
$$\mathbf{f}(\mathit{POLITICS}) = [0\ 0\ 0\ 0\ 1\ 0\ 1\ 0\ 0\ 0\ 0\ 0]$$

… *win the election* …

$$\mathbf{f}(\mathit{OTHER}) = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 1\ 0]$$

Non-Block Feature Vectors

- Sometimes the features of candidates cannot be decomposed in this regular way
- Example: a parse tree's features may be the productions present in the tree



- Different candidates will thus often share features
- We'll return to the non-block case later

Linear Models

Linear Models: Scoring

- In a linear model, each feature gets a weight w

… win the election …

$$\mathbf{f}(\textcolor{blue}{POLITICS}) = [\ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0]$$

… win the election …

$$\mathbf{f}(\textcolor{red}{SPORTS}) = [\ 1\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0]$$

$$\mathbf{w} = [\ 1\ \ 1\ \ -1\ \ -2\ \ 1\ \ -1\ \ 1\ \ -2\ \ -2\ \ -1\ \ -1\ \ 1]$$

- We score hypotheses by multiplying features and weights:

$$score(\mathbf{y}, \mathbf{w}) = \mathbf{w}^\top \mathbf{f}(\mathbf{y})$$

… win the election …

$$\mathbf{f}(\textcolor{blue}{POLITICS}) = [\ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0]$$

$$\mathbf{w} = [\ 1\ \ 1\ \ -1\ \ -2\ \ 1\ \ -1\ \ 1\ \ -2\ \ -2\ \ -1\ \ -1\ \ 1]$$

… win the election …

$$score(\textcolor{blue}{POLITICS}, \mathbf{w}) = 1 \times 1 + 1 \times 1 = 2$$

Linear Models: Decision Rule

- The linear decision rule:

$$prediction(\text{… win the election …}, \mathbf{w}) = \arg\max_{\mathbf{y} \in \mathcal{Y}(\mathbf{x})} \mathbf{w}^\top \mathbf{f}(\mathbf{y})$$

$$score(\overset{\text{… win the election …}}{SPORTS}, \mathbf{w}) = 1 \times 1 + (-1) \times 1 = 0$$

$$score(\overset{\text{… win the election …}}{POLITICS}, \mathbf{w}) = 1 \times 1 + 1 \times 1 = 2$$

$$score(\overset{\text{… win the election …}}{OTHER}, \mathbf{w}) = (-2) \times 1 + (-1) \times 1 = -3$$

$$prediction(\text{… win the election …}, \mathbf{w}) = \overset{\text{… win the election …}}{POLITICS}$$

- We've said nothing about where weights come from

Binary Classification

- Important special case: binary classification
  - Classes are y=+1/-1

$$\mathbf{f}(\mathbf{x}, -1) = -\mathbf{f}(\mathbf{x}, +1)$$

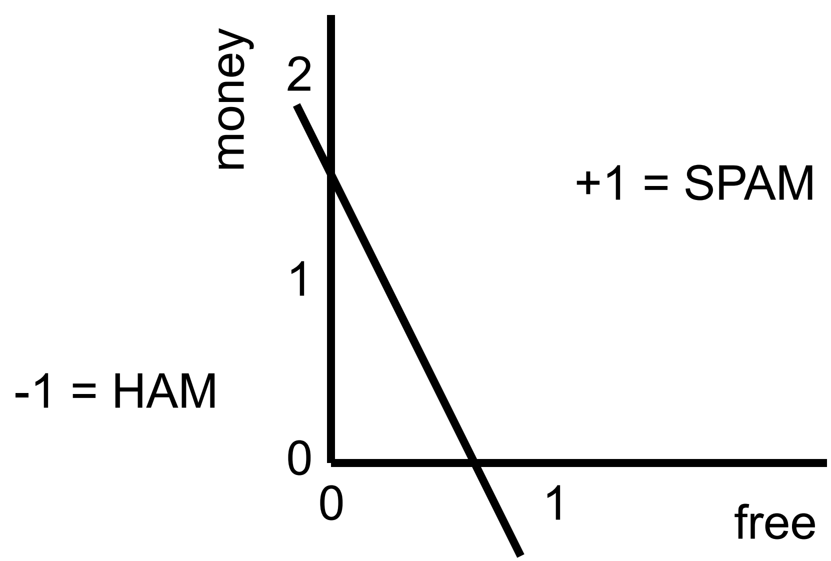
$$\mathbf{f}(\mathbf{x}) = 2\mathbf{f}(\mathbf{x}, +1)$$

  - Decision boundary is a hyperplane

$$\mathbf{w}^\top \mathbf{f}(\mathbf{x}) = 0$$

$\mathbf{w}$

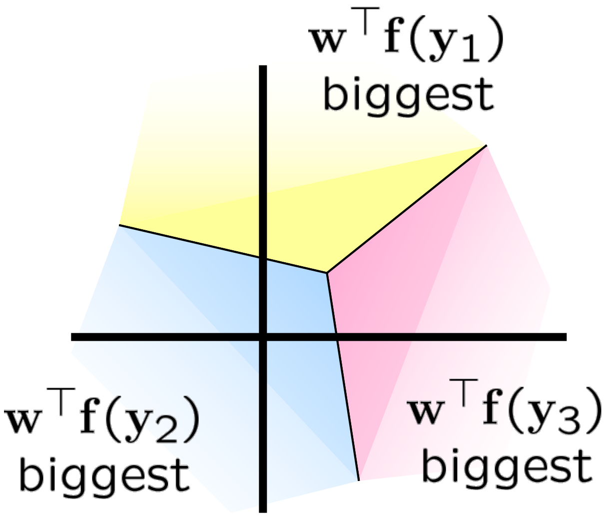
```
BIAS  : -3
free  :  4
money :  2
```



$$\mathbf{w}^\top \mathbf{f} = 0$$

Multiclass Decision Rule

- If more than two classes:
  - Highest score wins
  - Boundaries are more complex
  - Harder to visualize



$$prediction(\mathbf{x}_i, \mathbf{w}) = \arg\max_{\mathbf{y} \in \mathcal{Y}} \mathbf{w}^\top \mathbf{f}_i(\mathbf{y})$$

Learning

Learning Classifier Weights

- Two broad approaches to learning weights
- Generative: work with a probabilistic model of the data, weights are (log) local conditional probabilities
  - Advantages: learning weights is easy, smoothing is well-understood, backed by understanding of modeling
- Discriminative: set weights based on some error-related criterion
  - Advantages: error-driven, often weights which are good for classification aren't the ones which best describe the data
- We'll mainly talk about the latter for now

How to pick weights?

- Goal: choose "best" vector w given training data
  - For now, we mean "best for classification"
- The ideal: the weights which have greatest test set accuracy / F1 / whatever
  - But, don't have the test set
  - Must compute weights from training set
- Maybe we want weights which give best training set accuracy?
  - Hard discontinuous optimization problem
  - May not (does not) generalize to test set
  - Easy to overfit

Though, min-error training for MT does exactly this.

Minimize Training Error?

- A loss function declares how costly each mistake is

$$\ell_i(\mathbf{y}) = \ell(\mathbf{y}, \mathbf{y}_i^*)$$

  - E.g. 0 loss for correct label, 1 loss for wrong label
  - Can weight mistakes differently (e.g. false positives worse than false negatives or Hamming distance over structured labels)

- We could, in principle, minimize training loss:

$$\min_{\mathbf{w}} \sum_i \ell_i \left(\arg\max_{\mathbf{y}} \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) \right)$$

- This is a hard, discontinuous optimization problem

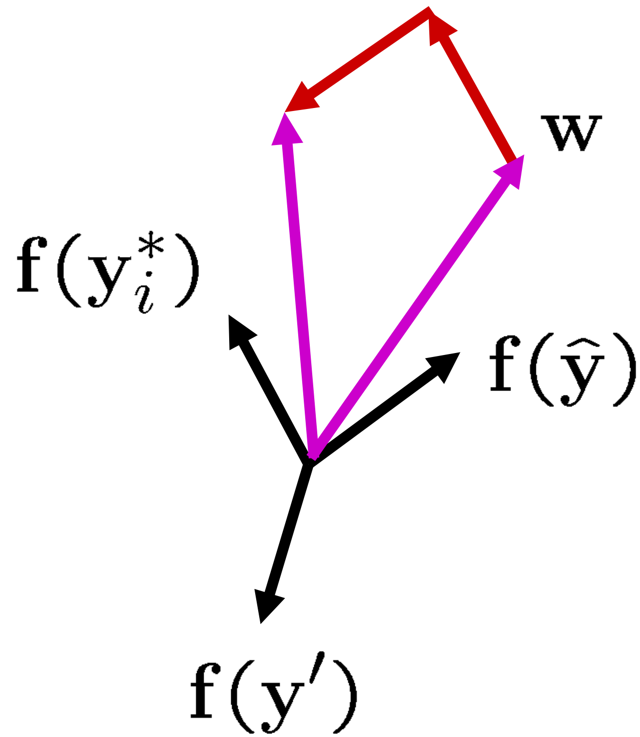
Linear Models: Perceptron

- The perceptron algorithm
  - Iteratively processes the training set, reacting to training errors
  - Can be thought of as trying to drive down training error
- The (online) perceptron algorithm:
  - Start with zero weights w
  - Visit training instances one by one
    - Try to classify

$$\hat{\mathbf{y}} = \arg\max_{\mathbf{y} \in \mathcal{Y}(\mathbf{x})} \mathbf{w}^\top \mathbf{f}(\mathbf{y})$$

    - If correct, no change!
    - If wrong: adjust weights

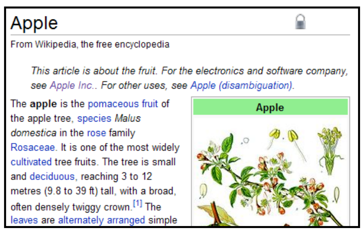
$$\mathbf{w} \leftarrow \mathbf{w} + \mathbf{f}(\mathbf{y}_i^*)$$
$$\mathbf{w} \leftarrow \mathbf{w} - \mathbf{f}(\hat{\mathbf{y}})$$



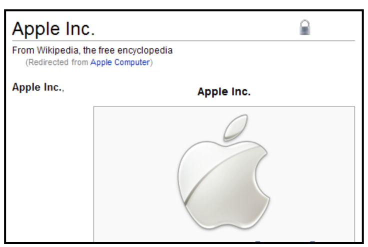
Example: "Best" Web Page

$$\mathbf{w} = [1 \quad 2 \quad 0 \quad 0 \quad \ldots]$$

x_i = "Apple Computers"

$\mathbf{f}_i($ 

$) = [0.3 \; 5 \; 0 \; 0 \; \ldots]$ $\qquad \mathbf{w}^\top \mathbf{f} = 10.3 \qquad \hat{\mathbf{y}}$

$\mathbf{f}_i($ 

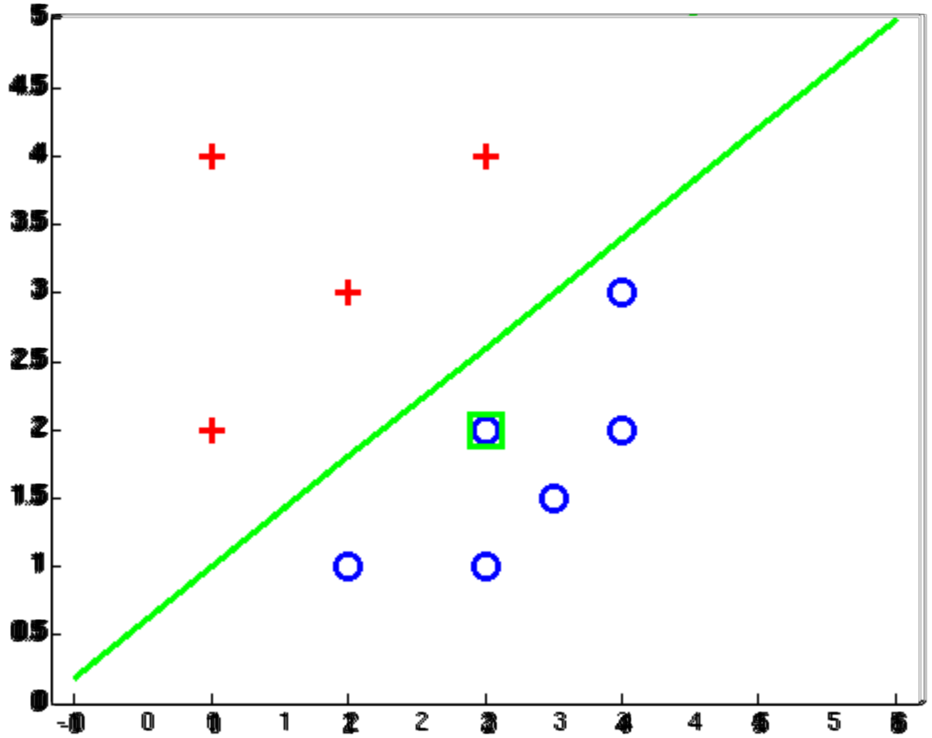
$) = [0.8 \; 4 \; 2 \; 1 \; \ldots]$ $\qquad \mathbf{w}^\top \mathbf{f} = 8.8 \qquad \mathbf{y}_i^*$

$$\mathbf{w} \leftarrow \mathbf{w} + \mathbf{f}(\mathbf{y}_i^*) - \mathbf{f}(\hat{\mathbf{y}})$$

$$\mathbf{w} = [1.5 \quad 1 \quad 2 \quad 1 \quad \ldots]$$

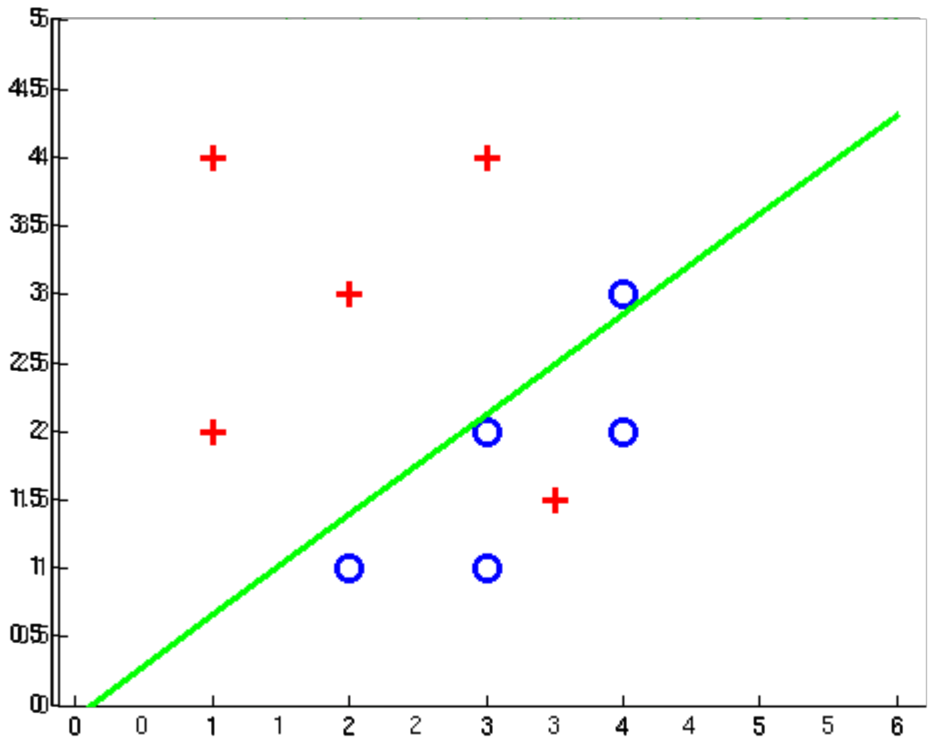
Examples: Perceptron

- Separable Case



Examples: Perceptron

- Non-Separable Case

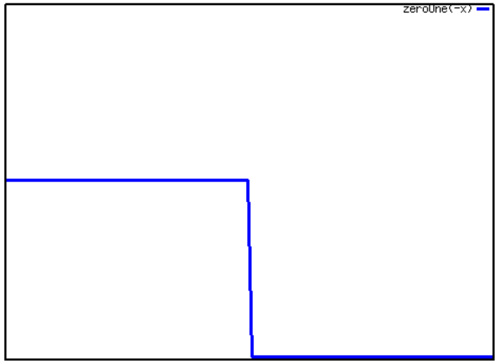


Margin

Objective Functions

- What do we want from our weights?
  - Depends!
  - So far: minimize (training) errors:

$$\sum_i step\left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) - \max_{\mathbf{y} \neq \mathbf{y}_i^*} \mathbf{w}^\top \mathbf{f}_i(\mathbf{y})\right)$$

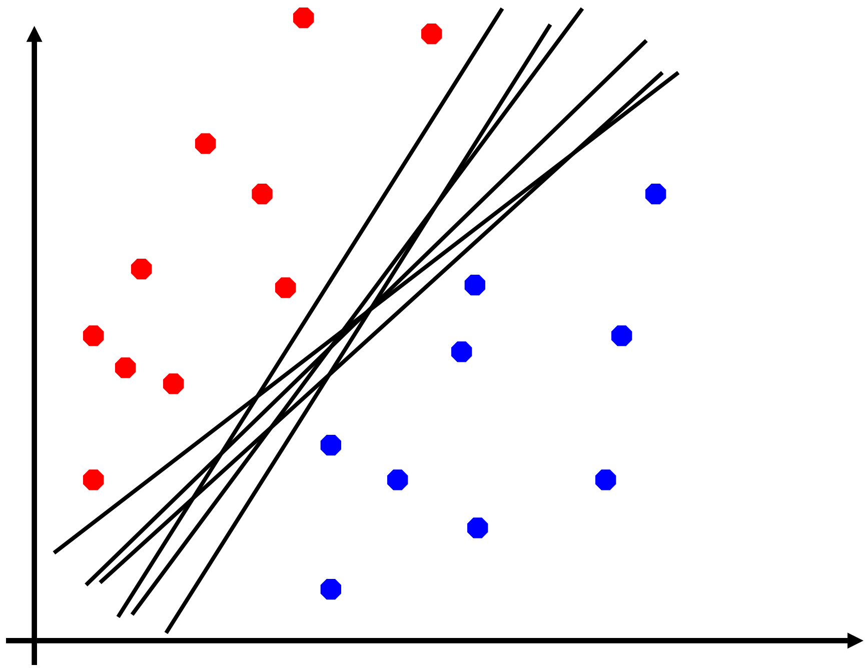


$$\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}^i) - \max_{\mathbf{y} \neq \mathbf{y}_i^*} \mathbf{w}^\top \mathbf{f}_i(\mathbf{y})$$

  - This is the “zero-one loss”
    - Discontinuous, minimizing is NP-complete
  - Maximum entropy and SVMs have other objectives related to zero-one loss

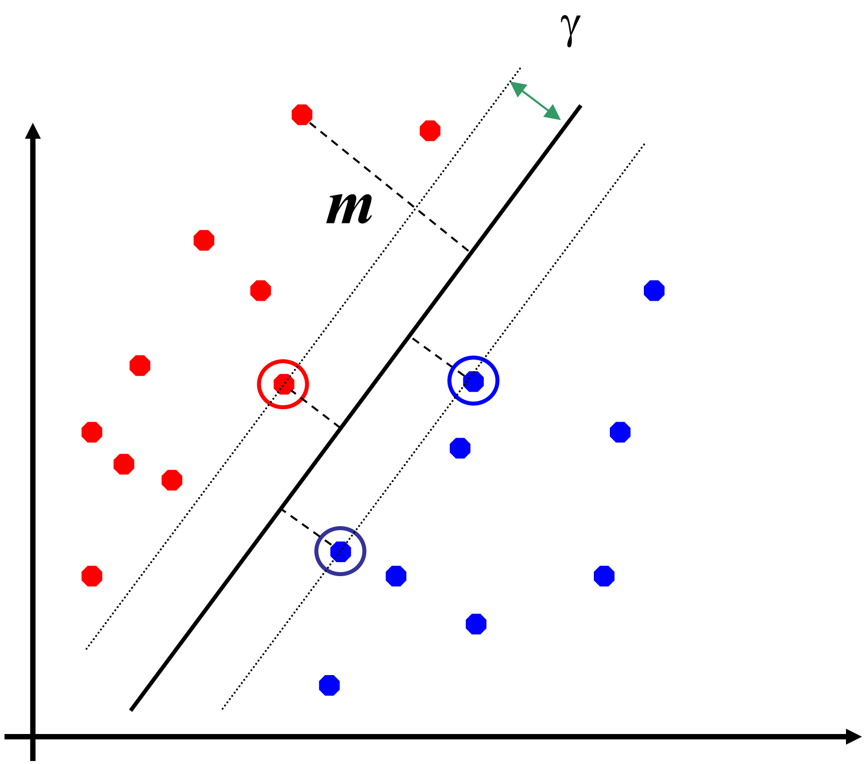
Linear Separators

- Which of these linear separators is optimal?



Classification Margin (Binary)

- Distance of $\mathbf{x}_i$ to separator is its margin, $\boldsymbol{m_i}$
- Examples closest to the hyperplane are support vectors
- Margin γ of the separator is the minimum $\boldsymbol{m}$



Classification Margin

- For each example $\mathbf{x}_i$ and possible mistaken candidate $\mathbf{y}$, we avoid that mistake by a margin $\boldsymbol{m_i}(\mathbf{y})$ (with zero-one loss)

$$m_i(\mathbf{y}) = \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) - \mathbf{w}^\top \mathbf{f}_i(\mathbf{y})$$

- Margin γ of the entire separator is the minimum $\boldsymbol{m}$

$$\gamma = \min_i \left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) - \max_{\mathbf{y} \neq \mathbf{y}_i^*} \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) \right)$$

- It is also the largest γ for which the following constraints hold

$$\forall i, \forall \mathbf{y} \quad \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) \geq \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) + \gamma \ell_i(\mathbf{y})$$

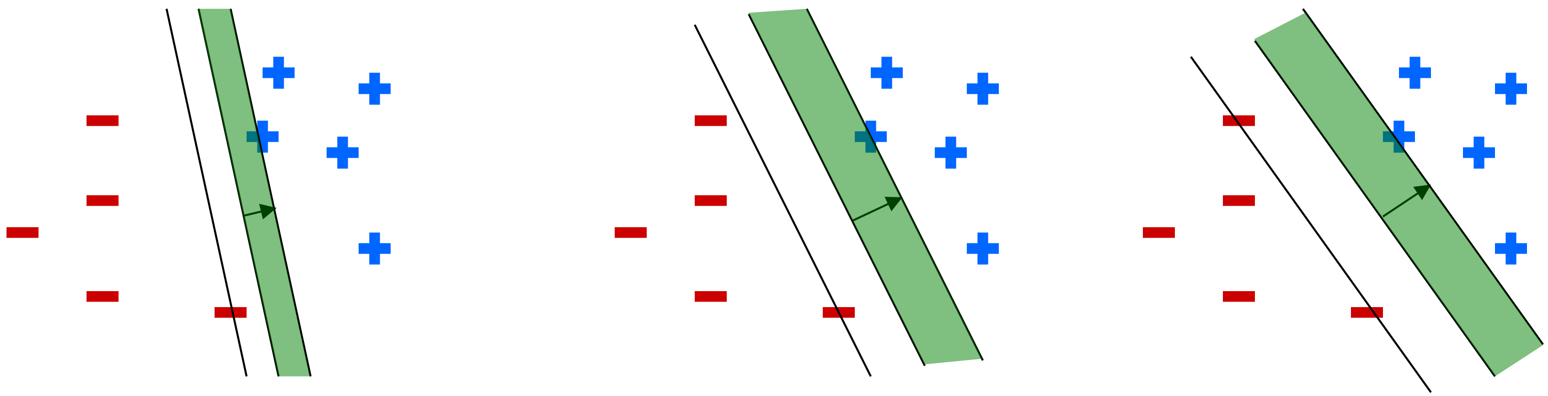
Maximum Margin

- Separable SVMs: find the max-margin w

$$\max_{||\mathbf{w}||=1} \gamma$$

$$\ell_i(\mathbf{y}) = \begin{cases} 0 & \text{if } \mathbf{y} = \mathbf{y}_i^* \\ 1 & \text{if } \mathbf{y} \neq \mathbf{y}_i^* \end{cases}$$

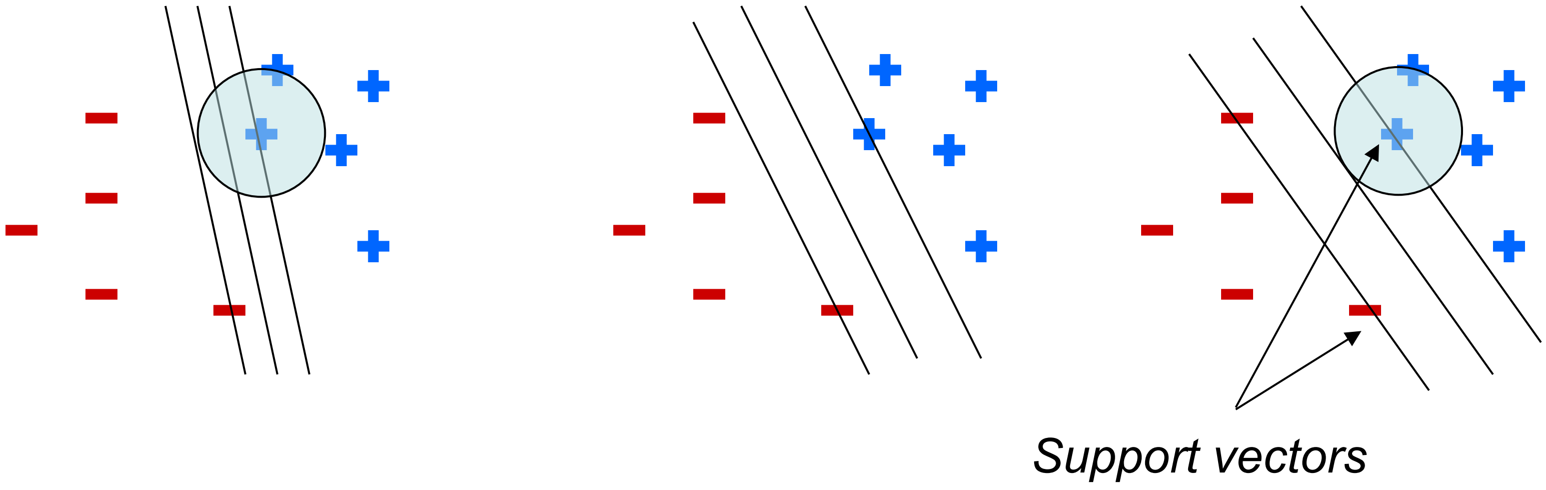
$$\forall i, \forall \mathbf{y} \quad \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) \geq \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) + \gamma \ell_i(\mathbf{y})$$



  - Can stick this into Matlab and (slowly) get an SVM
  - Won't work (well) if non-separable

Why Max Margin?

- Why do this? Various arguments:
  - Solution depends only on the boundary cases, or *support vectors* (but remember how this diagram is broken!)
  - Solution robust to movement of support vectors
  - Sparse solutions (features not in support vectors get zero weight)
  - Generalization bound arguments
  - Works well in practice for many problems



Max Margin / Small Norm

- Reformulation: find the smallest w which separates data

Remember this condition?

$$\max_{||\mathbf{w}||=1} \gamma$$

$$\forall i, \mathbf{y} \quad \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) \geq \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) + \gamma \ell_i(\mathbf{y})$$

- γ scales linearly in w, so if ||w|| isn't constrained, we can take any separating w and scale up our margin

$$\gamma = \min_{i, \mathbf{y} \neq \mathbf{y}_i^*} [\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) - \mathbf{w}^\top \mathbf{f}_i(\mathbf{y})] / \ell_i(\mathbf{y})$$

- Instead of fixing the scale of w, we can fix $\gamma = 1$

$$\min_{\mathbf{w}} \frac{1}{2} ||\mathbf{w}||^2$$

$$\forall i, \mathbf{y} \quad \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) \geq \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) + 1 \ell_i(\mathbf{y})$$

Gamma to w

$$\max_{||\mathbf{w}||=1} \gamma$$

$$\forall i, \mathbf{y} \quad \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) \geq \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) + \gamma \ell_i(\mathbf{y})$$

$$\mathbf{w} = \gamma u$$

$$\gamma = 1/||u||$$

$$\max_{||\gamma u||=1} 1/||u||^2$$

$$\forall i, \mathbf{y} \quad \gamma u^\top \mathbf{f}_i(\mathbf{y}_i^*) \geq \gamma u^\top \mathbf{f}_i(\mathbf{y}) + \gamma \ell_i(\mathbf{y})$$

$$\max_{||\gamma u||=1} 1/||u||^2$$

$$\forall i, \mathbf{y} \quad u^\top \mathbf{f}_i(\mathbf{y}_i^*) \geq u^\top \mathbf{f}_i(\mathbf{y}) + \ell_i(\mathbf{y})$$

$$\min_{||\gamma u||=1} ||u||^2$$

$$\forall i, \mathbf{y} \quad u^\top \mathbf{f}_i(\mathbf{y}_i^*) \geq u^\top \mathbf{f}_i(\mathbf{y}) + \ell_i(\mathbf{y})$$

$$\min_{u} ||u||^2$$

$$\forall i, \mathbf{y} \quad u^\top \mathbf{f}_i(\mathbf{y}_i^*) \geq u^\top \mathbf{f}_i(\mathbf{y}) + \ell_i(\mathbf{y})$$

$$\min_{u} \frac{1}{2}||u||^2$$

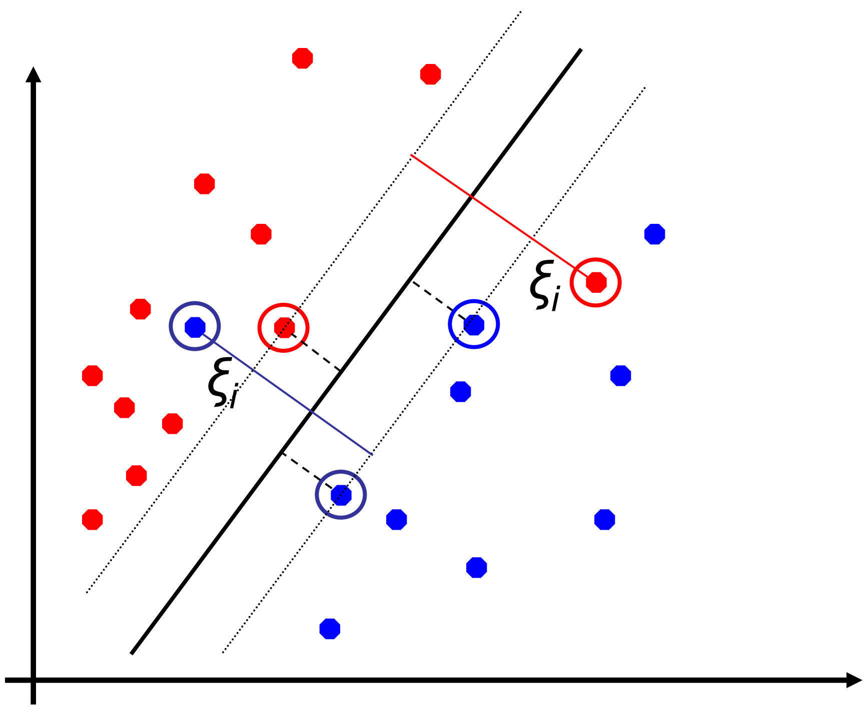
$$\forall i, \mathbf{y} \quad u^\top \mathbf{f}_i(\mathbf{y}_i^*) \geq u^\top \mathbf{f}_i(\mathbf{y}) + \ell_i(\mathbf{y})$$

$$\min_{\mathbf{w}} \frac{1}{2}||\mathbf{w}||^2$$

$$\forall i, \mathbf{y} \quad \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) \geq \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) + \ell_i(\mathbf{y})$$

Soft Margin Classification

- What if the training set is not linearly separable?
- *Slack variables* $\boldsymbol{\xi_i}$ can be added to allow misclassification of difficult or noisy examples, resulting in a *soft margin* classifier



Maximum Margin

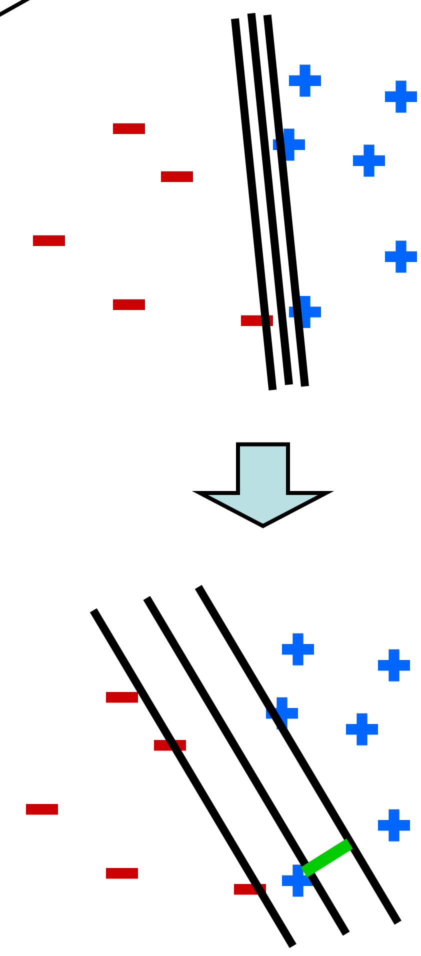
Note: exist other choices of how to penalize slacks!

- Non-separable SVMs
  - Add slack to the constraints
  - Make objective pay (linearly) for slack:

$$\min_{\mathbf{w},\xi} \frac{1}{2}||\mathbf{w}||^2 + C\sum_i \xi_i$$

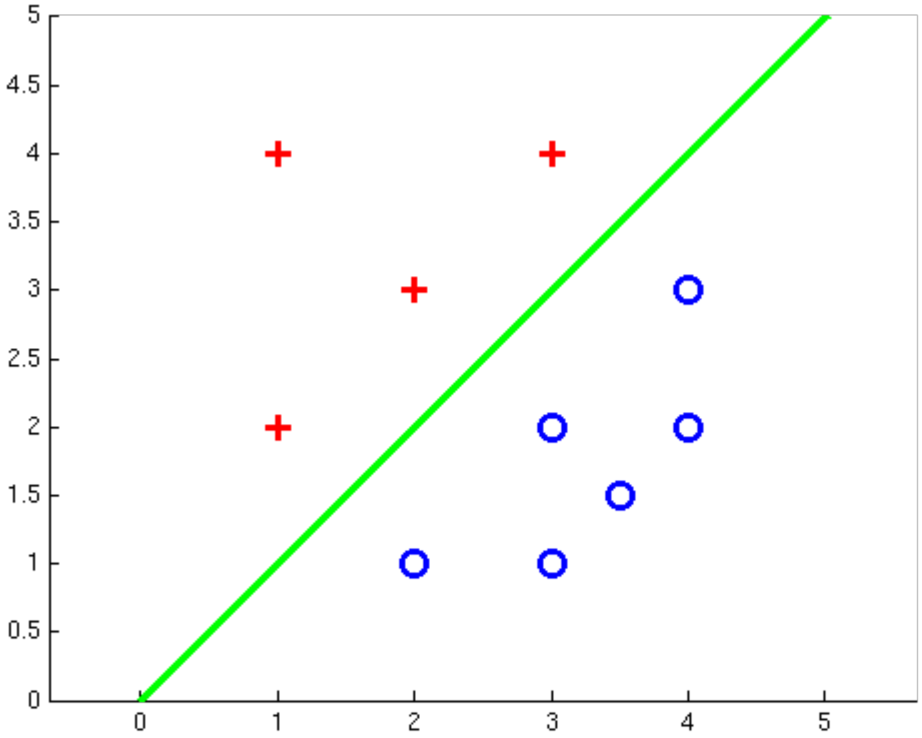
$$\forall i, \mathbf{y}, \quad \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) + \xi_i \geq \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) + \ell_i(\mathbf{y})$$

  - C is called the *capacity* of the SVM – the smoothing knob



- Learning:
  - Can still stick this into Matlab if you want
  - Constrained optimization is hard; better methods!
  - We'll come back to this later

Maximum Margin



Likelihood

Linear Models: Maximum Entropy

- Maximum entropy (logistic regression)
  - Use the scores as probabilities:

$$\mathsf{P}(\mathbf{y}|\mathbf{x},\mathbf{w}) = \frac{\exp(\mathbf{w}^\top \mathbf{f}(\mathbf{y}))}{\sum_{\mathbf{y}'} \exp(\mathbf{w}^\top \mathbf{f}(\mathbf{y}'))}$$

Make positive

Normalize

  - Maximize the (log) conditional likelihood of training data

$$L(\mathbf{w}) = \log \prod_i \mathsf{P}(\mathbf{y}_i^*|\mathbf{x}_i,\mathbf{w}) = \sum_i \log \left(\frac{\exp(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*))}{\sum_{\mathbf{y}} \exp(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}))} \right)$$

$$= \sum_i \left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) - \log \sum_{\mathbf{y}} \exp(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y})) \right)$$

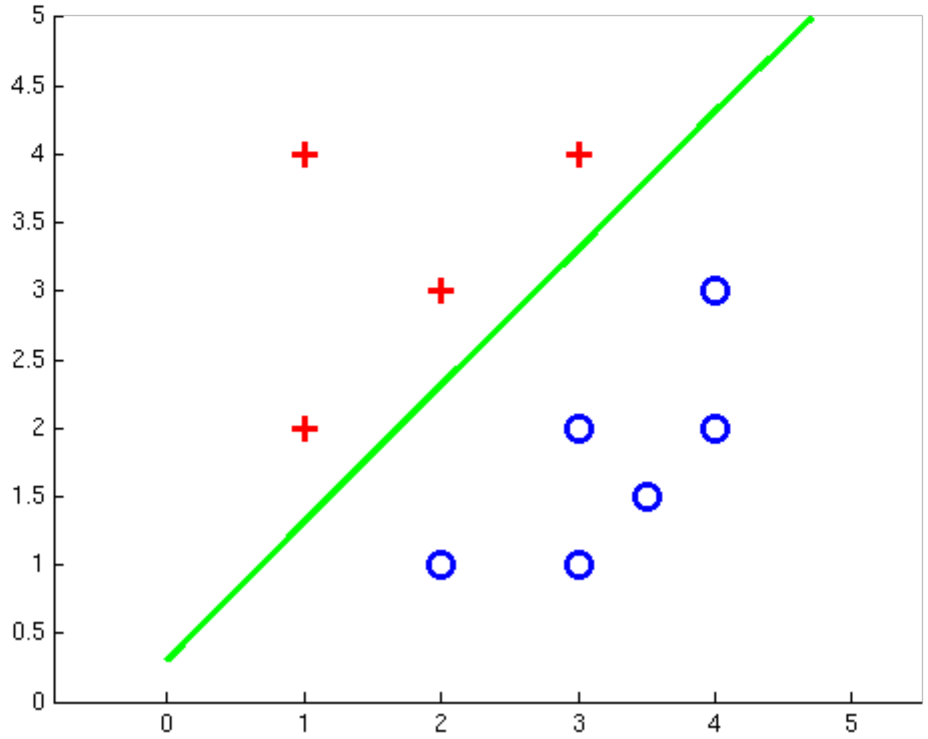
Maximum Entropy II

- Motivation for maximum entropy:
  - Connection to maximum entropy principle (sort of)
  - Might want to do a good job of being uncertain on noisy cases…
  - … in practice, though, posteriors are pretty peaked
- Regularization (smoothing)

$$\max_{\mathbf{w}} \sum_i \left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) - \log \sum_{\mathbf{y}} \exp(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y})) \right) - k||\mathbf{w}||^2$$

$$\min_{\mathbf{w}} \; k||\mathbf{w}||^2 - \sum_i \left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) - \log \sum_{\mathbf{y}} \exp(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y})) \right)$$

Maximum Entropy



Loss Comparison

Log-Loss

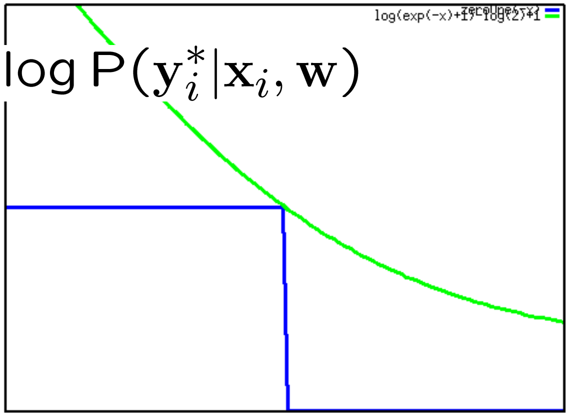
- If we view maxent as a minimization problem:

$$\min_{\mathbf{w}} \quad k||\mathbf{w}||^2 + \sum_i -\left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) - \log \sum_{\mathbf{y}} \exp(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}))\right)$$

- This minimizes the “log loss” on each example

$$-\left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) - \log \sum_{\mathbf{y}} \exp(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}))\right) = -\log \mathrm{P}(\mathbf{y}_i^* | \mathbf{x}_i, \mathbf{w})$$

$$step\left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) - \max_{\mathbf{y} \neq \mathbf{y}_i^*} \mathbf{w}^\top \mathbf{f}_i(\mathbf{y})\right)$$



- One view: log loss is an *upper bound* on zero-one loss

Remember SVMs…

- We had a constrained minimization

$$\min_{\mathbf{w},\xi} \frac{1}{2}||\mathbf{w}||^2 + C\sum_i \xi_i$$

$$\forall i, \mathbf{y}, \quad \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) + \xi_i \geq \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) + \ell_i(\mathbf{y})$$

- …but we can solve for ξ_i

$$\forall i, \mathbf{y}, \quad \xi_i \geq \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) + \ell_i(\mathbf{y}) - \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*)$$

$$\forall i, \quad \xi_i = \max_{\mathbf{y}} \left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) + \ell_i(\mathbf{y})\right) - \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*)$$

- Giving

$$\min_{\mathbf{w}} \frac{1}{2}||\mathbf{w}||^2 + C\sum_i \left(\max_{\mathbf{y}} \left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) + \ell_i(\mathbf{y})\right) - \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*)\right)$$

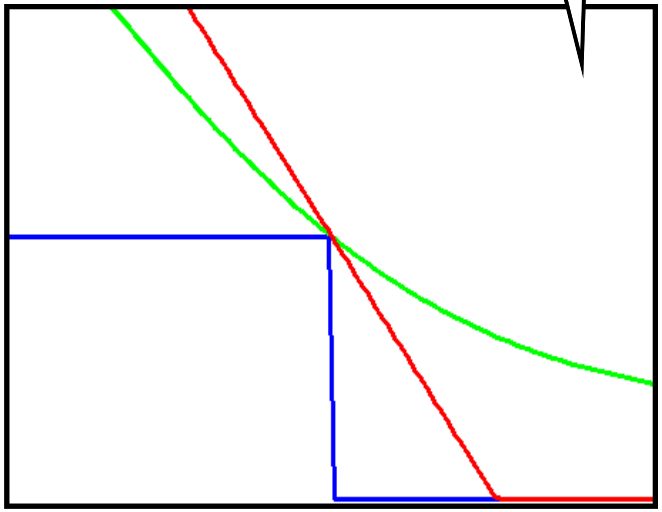
Hinge Loss

Plot really only right in binary case

- Consider the per-instance objective:

$$\min_{\mathbf{w}} \; k||\mathbf{w}||^2 + \sum_i \left(\max_{\mathbf{y}} \left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) + \ell_i(y) \right) - \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) \right)$$

- This is called the "hinge loss"
  - Unlike maxent / log loss, you stop gaining objective once the true label wins by enough
  - You can start from here and derive the SVM objective
  - Can solve directly with sub-gradient decent (e.g. Pegasos: Shalev-Shwartz et al 07)



$$\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) - \max_{\mathbf{y} \neq \mathbf{y}_i^*} \left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) \right)$$

Max vs "Soft-Max" Margin

- SVMs:

$$\min_{\mathbf{w}} \quad k||\mathbf{w}||^2 - \sum_i \left(\underbrace{\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) - \max_{\mathbf{y}} \left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) + \ell_i(y) \right)}_{\text{You can make this zero}} \right)$$

- Maxent:

$$\min_{\mathbf{w}} \quad k||\mathbf{w}||^2 - \sum_i \left(\underbrace{\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) - \log \sum_{\mathbf{y}} \exp \left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) \right)}_{\text{... but not this one}} \right)$$

- Very similar! Both try to make the true score better than a function of the other scores
  - The SVM tries to beat the augmented runner-up
  - The Maxent classifier tries to beat the "soft-max"

Loss Functions: Comparison

- Zero-One Loss

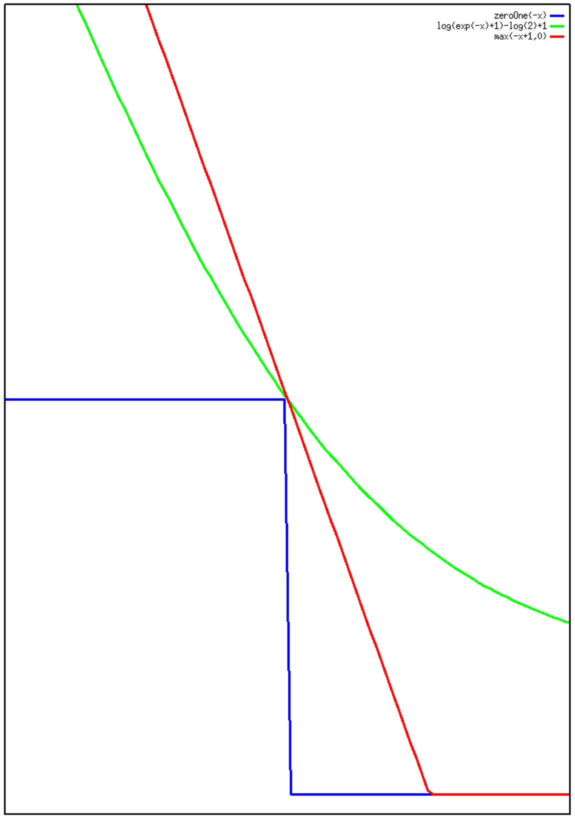
$$\sum_i step\left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) - \max_{\mathbf{y} \neq \mathbf{y}_i^*} \mathbf{w}^\top \mathbf{f}_i(\mathbf{y})\right)$$

- Hinge

$$\sum_i \left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) - \max_{\mathbf{y}} \left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) + \ell_i(y)\right)\right)$$

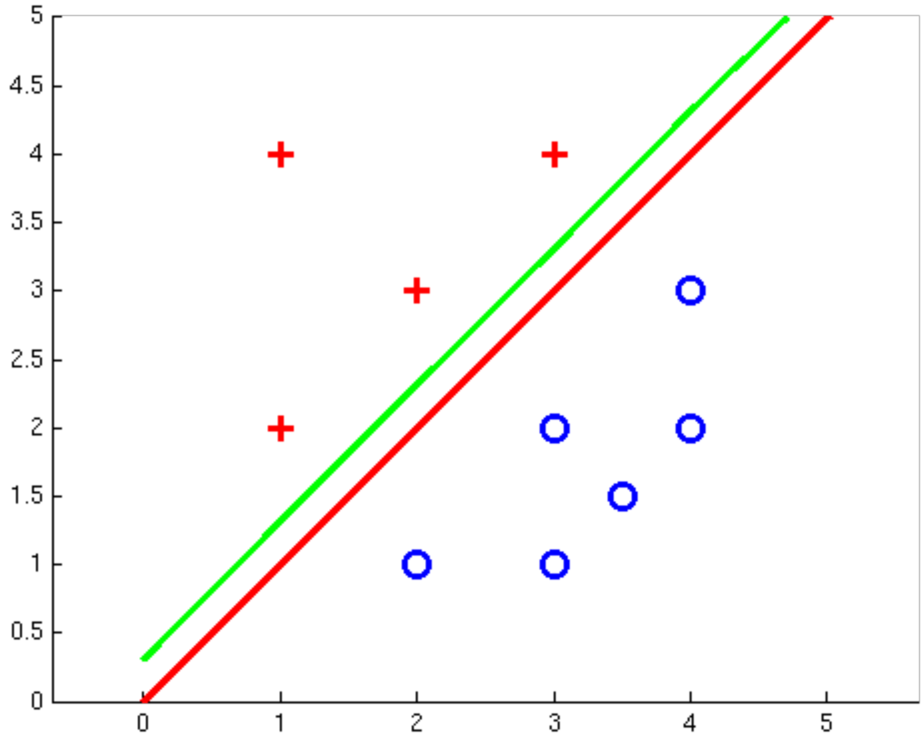
- Log

$$\sum_i \left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) - \log \sum_{\mathbf{y}} \exp\left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y})\right)\right)$$



$$\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) - \max_{\mathbf{y} \neq \mathbf{y}_i^*} \left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y})\right)$$

Separators: Comparison

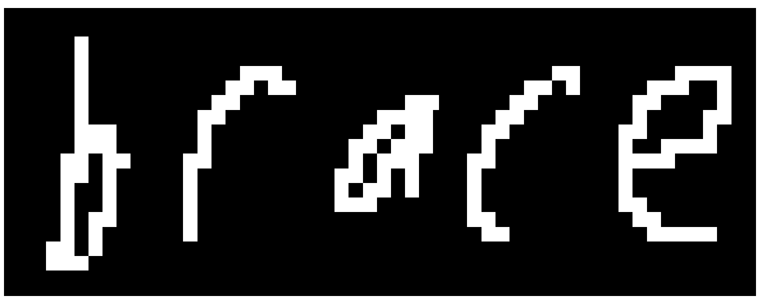


Structure

Handwriting recognition

x

y



→ **brace**

Sequential structure

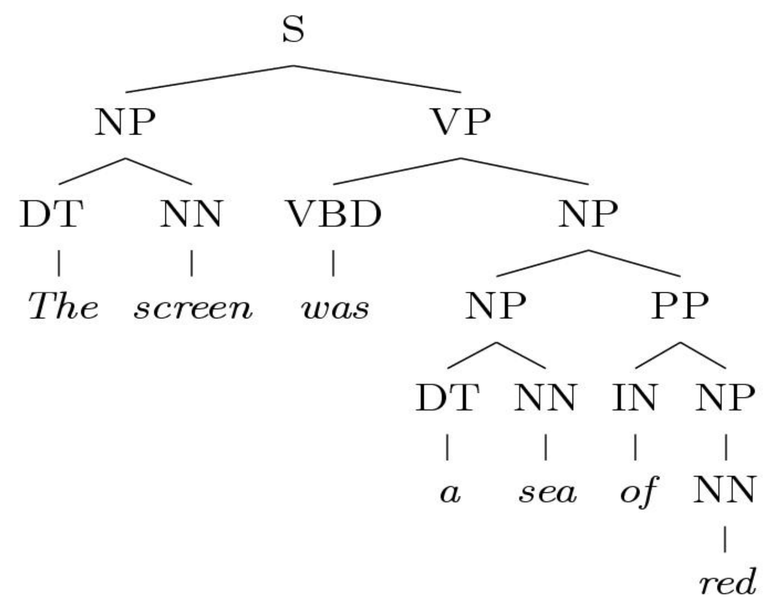
[Slides: Taskar and Klein 05]

CFG Parsing

x

The screen was a sea of red

y



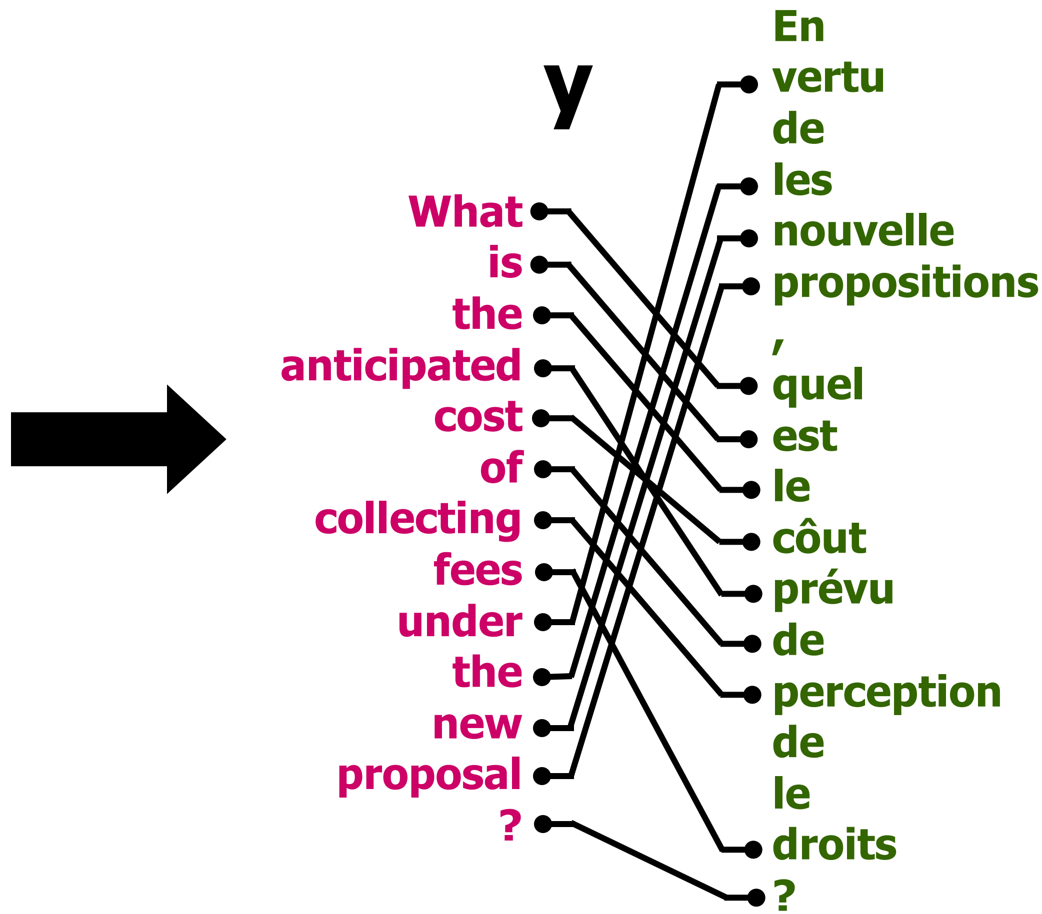
Recursive structure

Bilingual Word Alignment

x

What is the anticipated cost of collecting fees under the new proposal?

En vertu de nouvelle propositions, quel est le côut prévu de perception de les droits?



Combinatorial structure

Structured Models

$$prediction(\mathbf{x}, \mathbf{w}) = \arg\max_{\mathbf{y} \in \mathcal{Y}(\mathbf{x})} score(\mathbf{y}, \mathbf{w})$$

space of feasible outputs

Assumption:

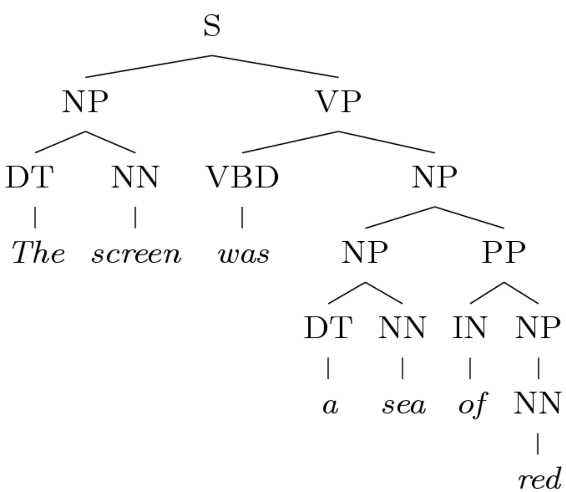
$$score(\mathbf{y}, \mathbf{w}) = \mathbf{w}^\top \mathbf{f}(\mathbf{y}) = \sum_p \mathbf{w}^\top \mathbf{f}(\mathbf{y}_p)$$

Score is a sum of local "part" scores

Parts = nodes, edges, productions

CFG Parsing

$$P(\mathbf{y} \mid \mathbf{x}) \propto \prod_{A \to \alpha \in (\mathbf{x},\mathbf{y})} \phi(A \to \alpha)$$



$$\mathbf{f} : \mathcal{X} \times \mathcal{Y} \to \Re^d$$

#(NP → DT NN)

…

#(PP → IN NP)

…

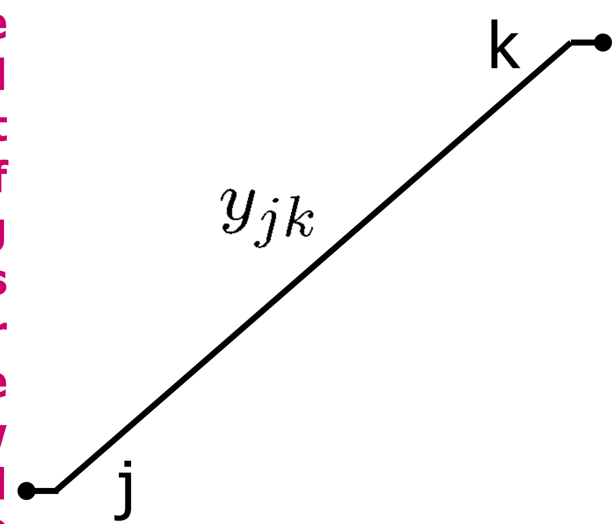
#(NN → ‘sea’)

$$\prod_{A \to \alpha \in (\mathbf{x},\mathbf{y})} \exp\left\{\mathbf{w}^\top \mathbf{f}(A \to \alpha)\right\} = \exp\{\mathbf{w}^\top \mathbf{f}(\mathbf{x},\mathbf{y})\}$$

Bilingual word alignment

$$\sum_{y_{jk} \in \mathbf{y}} \mathbf{w}^\top \mathbf{f}(\mathbf{x}_{jk}) = \mathbf{w}^\top \mathbf{f}(\mathbf{x}, \mathbf{y})$$

What
is
the
anticipated
cost
of
collecting
fees
under
the
new
proposal
?



En
vertu
de
les
nouvelle
propositions
,
quel
est
le
côut
prévu
de
perception
de
le
droits
?

$\mathbf{f}(\mathbf{x}_{jk})$

- association
- position
- orthography

Efficient Decoding

- Common case: you have a black box which computes

$$\text{prediction}(\mathbf{x}) = \arg\max_{\mathbf{y} \in \mathcal{Y}(\mathbf{x})} \mathbf{w}^\top \mathbf{f}(\mathbf{y})$$

  at least approximately, and you want to learn w

- Easiest option is the structured perceptron [Collins 01]
  - Structure enters here in that the search for the best y is typically a combinatorial algorithm (dynamic programming, matchings, ILPs, A*…)
  - Prediction is structured, learning update is not

Structured Margin (Primal)

Remember our primal margin objective?

$$\min_{w} \quad \frac{1}{2}\|w\|_2^2 + C\sum_i \left(\max_y \left(w^\top f_i(y) + \ell_i(y) \right) - w^\top f_i(y_i^*) \right)$$

Still applies with structured output space!

Structured Margin (Primal)

Just need efficient loss-augmented decode:

$$\bar{y} = \text{argmax}_y \left(w^\top f_i(y) + \ell_i(y) \right)$$

$$\min_w \quad \frac{1}{2}\|w\|_2^2 + C \sum_i \left(w^\top f_i(\bar{y}) + \ell_i(\bar{y}) - w^\top f_i(y_i^*) \right)$$

$$\nabla_w = w + C \sum_i \left(f_i(\bar{y}) - f_i(y_i^*) \right)$$

Still use general subgradient descent methods! (Adagrad)

Structured Margin (Dual)

- Remember the constrained version of primal:

$$\min_{\mathbf{w},\xi} \quad \frac{1}{2}||\mathbf{w}||^2 + C\sum_i \xi_i$$

$$\forall i, \mathbf{y} \quad \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) \geq \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) + \ell_i(\mathbf{y}) - \xi_i$$

- Dual has a variable for every constraint here

Full Margin: OCR

- We want:

$\arg\max_{\mathbf{y}} \ \mathbf{w}^\top \mathbf{f}($  $, \mathbf{y}) \ = \$ "brace"

- Equivalently:

$\mathbf{w}^\top \mathbf{f}($  , "brace"$) \ > \ \mathbf{w}^\top \mathbf{f}($  , "aaaaa"$)$

$\mathbf{w}^\top \mathbf{f}($  , "brace"$) \ > \ \mathbf{w}^\top \mathbf{f}($  , "aaaab"$)$

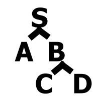
$\ldots$

$\mathbf{w}^\top \mathbf{f}($  , "brace"$) \ > \ \mathbf{w}^\top \mathbf{f}($  , "zzzzz"$)$

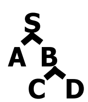
a lot!

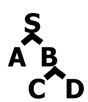
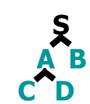
Parsing example

- We want:

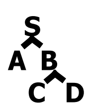
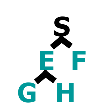
$$\arg\max_{\mathbf{y}} \ \mathbf{w}^\top \mathbf{f}(\text{'It was red'}, \mathbf{y}) =$$ 

- Equivalently:

$\mathbf{w}^\top \mathbf{f}(\text{'It was red'},$  $) > \mathbf{w}^\top \mathbf{f}(\text{'It was red'},$  $)$

$\mathbf{w}^\top \mathbf{f}(\text{'It was red'},$  $) > \mathbf{w}^\top \mathbf{f}(\text{'It was red'},$  $)$

...

$\mathbf{w}^\top \mathbf{f}(\text{'It was red'},$  $) > \mathbf{w}^\top \mathbf{f}(\text{'It was red'},$  $)$

a lot!

Alignment example

- We want:

$$\arg\max_{y} \; w^\top f\left(\begin{array}{l}\text{'What is the'}\\ \text{'Quel est le'}\end{array}, y\right) = \begin{array}{l} 1 \leftrightarrow 1 \\ 2 \leftrightarrow 2 \\ 3 \leftrightarrow 3 \end{array}$$

- Equivalently:

$$\left.\begin{array}{l}
w^\top f\left(\begin{array}{l}\text{'What is the'}\\ \text{'Quel est le'}\end{array}, \begin{array}{l} 1 \leftrightarrow 1 \\ 2 \leftrightarrow 2 \\ 3 \leftrightarrow 3 \end{array}\right) > w^\top f\left(\begin{array}{l}\text{'What is the'}\\ \text{'Quel est le'}\end{array}, \begin{array}{l} 1 \leftrightarrow 1 \\ 2 \leftrightarrow 3 \\ 3 \leftrightarrow 2 \end{array}\right) \\
w^\top f\left(\begin{array}{l}\text{'What is the'}\\ \text{'Quel est le'}\end{array}, \begin{array}{l} 1 \leftrightarrow 1 \\ 2 \leftrightarrow 2 \\ 3 \leftrightarrow 3 \end{array}\right) > w^\top f\left(\begin{array}{l}\text{'What is the'}\\ \text{'Quel est le'}\end{array}, \begin{array}{l} 1 \leftrightarrow 2 \\ 2 \leftrightarrow 1 \\ 3 \leftrightarrow 3 \end{array}\right) \\
\qquad\qquad \cdots \\
w^\top f\left(\begin{array}{l}\text{'What is the'}\\ \text{'Quel est le'}\end{array}, \begin{array}{l} 1 \leftrightarrow 1 \\ 2 \leftrightarrow 2 \\ 3 \leftrightarrow 3 \end{array}\right) > w^\top f\left(\begin{array}{l}\text{'What is the'}\\ \text{'Quel est le'}\end{array}, \begin{array}{l} 1 \leftrightarrow 3 \\ 2 \leftrightarrow 2 \\ 3 \leftrightarrow 1 \end{array}\right)
\end{array}\right\} \text{a lot!}$$

Cutting Plane (Dual)

- A constraint induction method [Joachims et al 09]
  - Exploits that the number of constraints you actually need per instance is typically very small
  - Requires (loss-augmented) primal-decode only

- Repeat:
  - Find the most violated constraint for an instance:

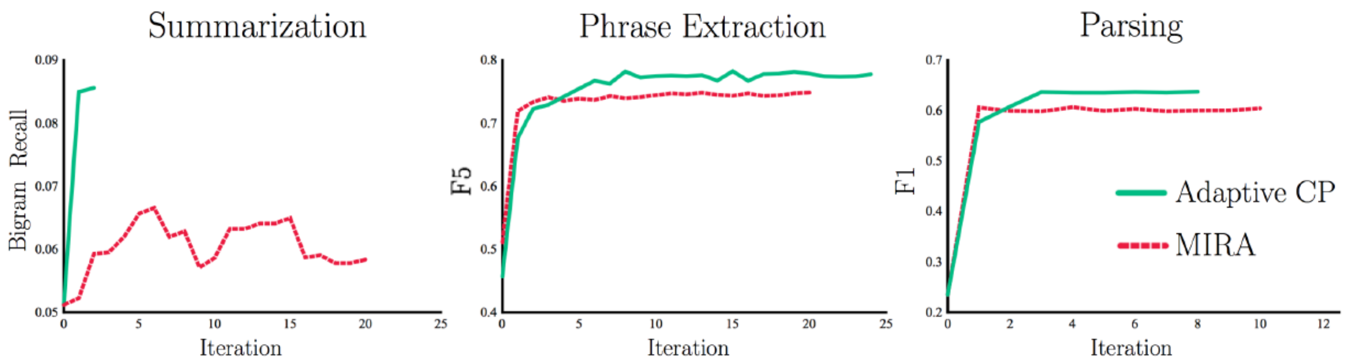
$$\forall \mathbf{y} \quad \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) \geq \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) + \ell_i(\mathbf{y})$$

$$\arg\max_{\mathbf{y}} \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) + \ell_i(\mathbf{y})$$

  - Add this constraint and resolve the (non-structured) QP (e.g. with SMO or other QP solver)

Cutting Plane (Dual)

- Some issues:
  - Can easily spend too much time solving QPs
  - Doesn't exploit shared constraint structure
  - In practice, works pretty well; fast like perceptron/MIRA, more stable, no averaging



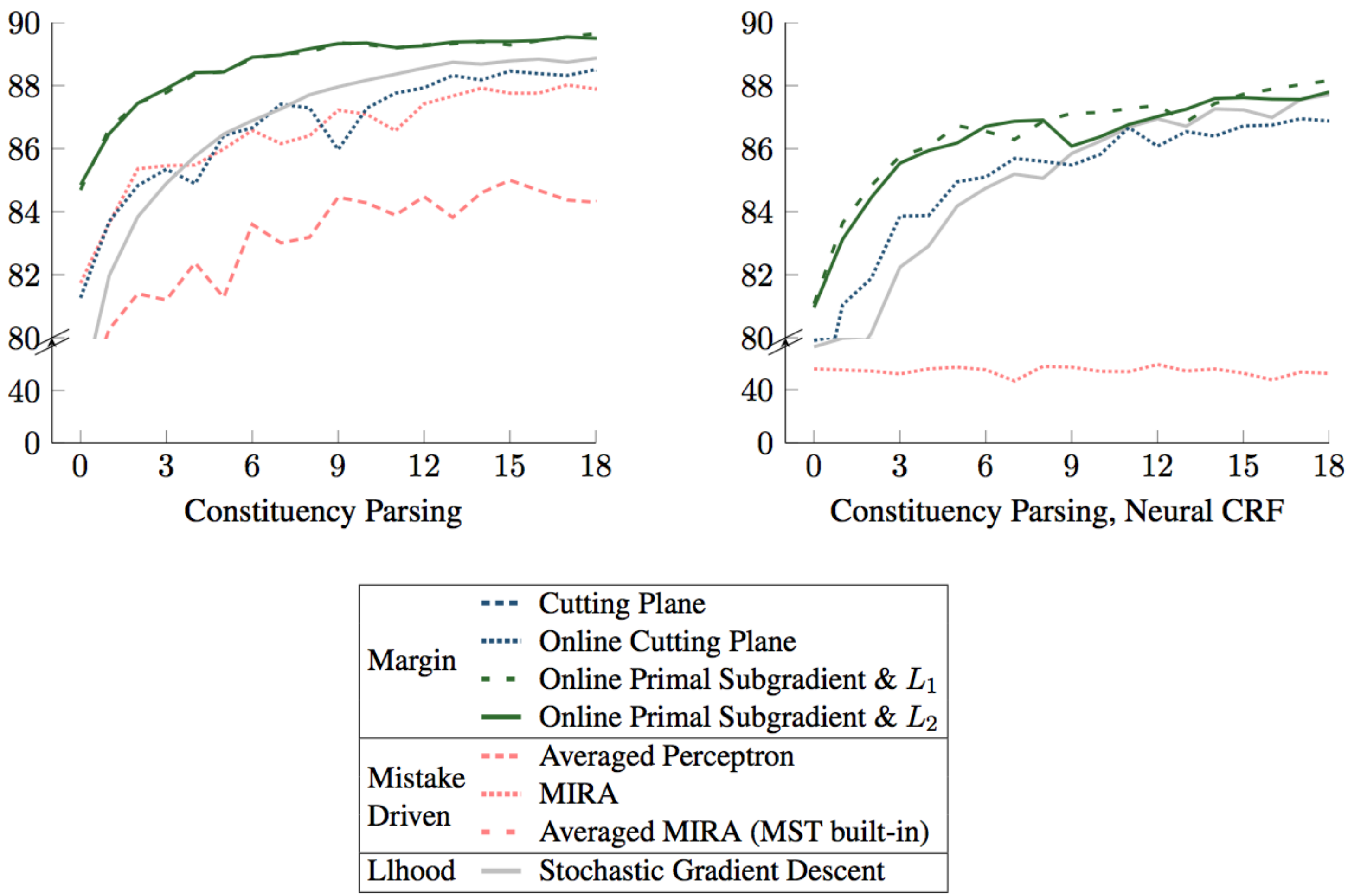
Likelihood, Structured

$$L(\mathbf{w}) = -k||\mathbf{w}||^2 + \sum_i \left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) - \log \sum_{\mathbf{y}} \exp(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y})) \right)$$

$$\frac{\partial L(\mathbf{w})}{\partial \mathbf{w}} = -2k\mathbf{w} + \sum_i \left(\mathbf{f}_i(\mathbf{y}_i^*) - \sum_{\mathbf{y}} P(\mathbf{y}|\mathbf{x}_i)\mathbf{f}_i(\mathbf{y}) \right)$$

- Structure needed to compute:
  - Log-normalizer
  - Expected feature counts
    - E.g. if a feature is an indicator of DT-NN then we need to compute posterior marginals P(DT-NN|sentence) for each position and sum
- Also works with latent variables (more later)

Comparison



90
88
86
84
82
80
40
0
0 3 6 9 12 15 18
Constituency Parsing
Constituency Parsing, Neural CRF
Margin
Cutting Plane
Online Cutting Plane
Online Primal Subgradient & L_1
Online Primal Subgradient & L_2
Mistake Driven
Averaged Perceptron
MIRA
Averaged MIRA (MST built-in)
Llhood
Stochastic Gradient Descent

Option 0: Reranking

[e.g. Charniak and Johnson 05]

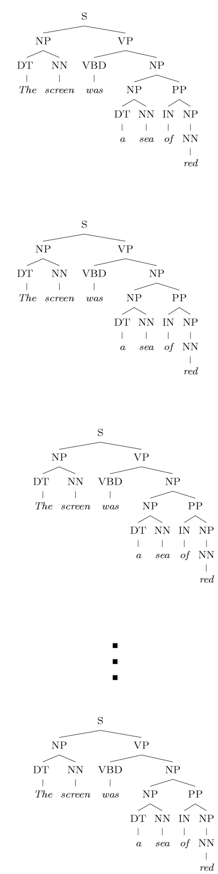
Input

N-Best List
(e.g. n=100)

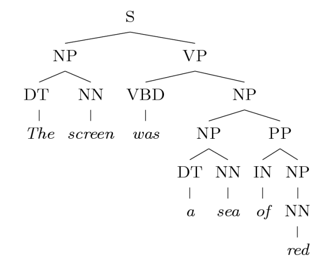
Output

x =
“The screen was a sea of red.”

Baseline Parser

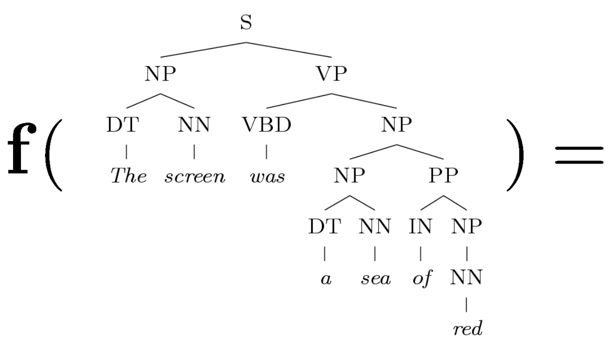


Non-Structured Classification



Reranking

- Advantages:
  - Directly reduce to non-structured case
  - No locality restriction on features



- Disadvantages:
  - Stuck with errors of baseline parser
  - Baseline system must produce n-best lists
  - But, feedback is possible [McCloskey, Charniak, Johnson 2006]

M3Ns

- Another option: express all constraints in a packed form
  - Maximum margin Markov networks [Taskar et al 03]
  - Integrates solution structure deeply into the problem structure

- Steps
  - Express inference over constraints as an LP
  - Use duality to transform minimax formulation into min-min
  - Constraints factor in the dual along the same structure as the primal; alphas essentially act as a dual “distribution”
  - Various optimization possibilities in the dual

Example: Kernels

- Quadratic kernels

$$K(x, x') = (x \cdot x' + 1)^2$$

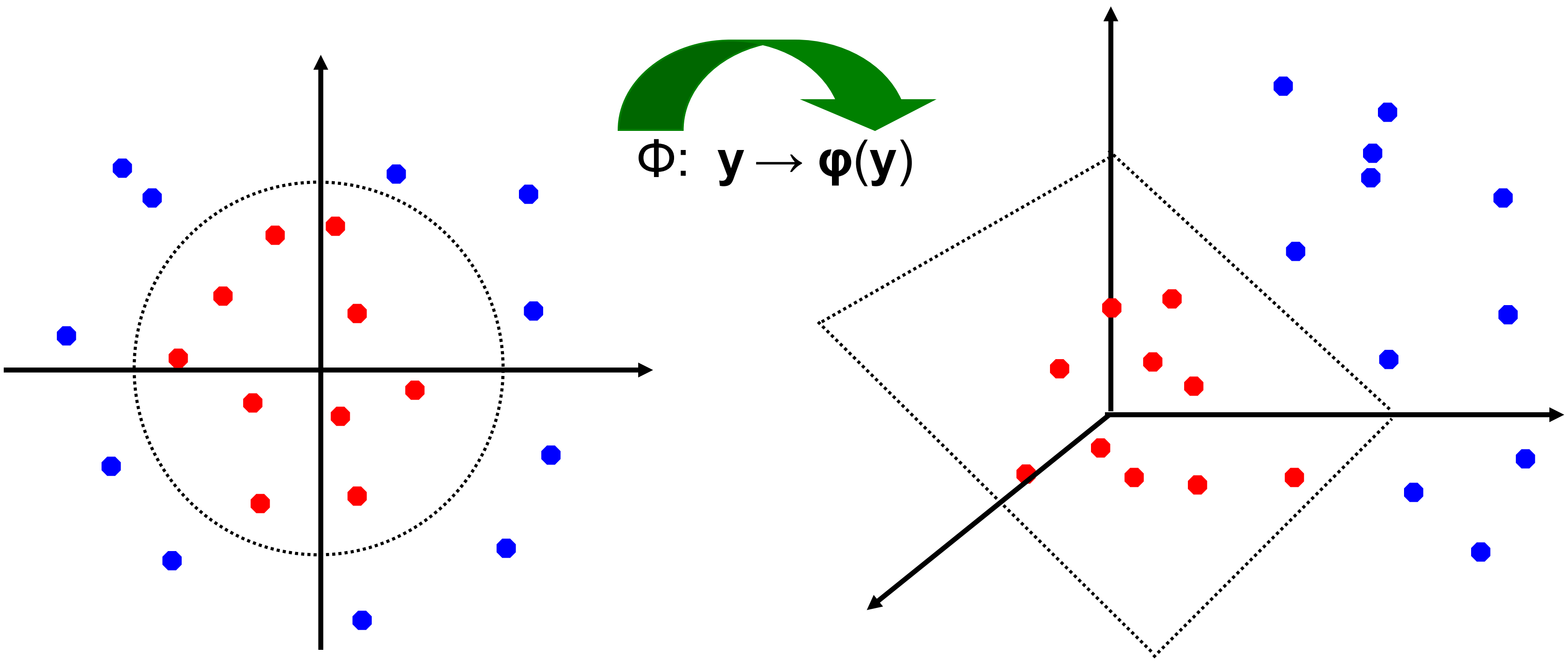
$$= \sum_{i,j} x_i x_j \, x'_i x'_j + 2 \sum_i x_i \, x'_i + 1$$

$$\Downarrow$$

$$K(\mathbf{y}, \mathbf{y}') = (\mathbf{f}(\mathbf{y})^\top \mathbf{f}(\mathbf{y}') + 1)^2$$

Non-Linear Separators

- Another view: kernels map an original feature space to some higher-dimensional feature space where the training set is (more) separable



Why Kernels?

- Can’t you just add these features on your own (e.g. add all pairs of features instead of using the quadratic kernel)?
  - Yes, in principle, just compute them
  - No need to modify any algorithms
  - But, number of features can get large (or infinite)
  - Some kernels not as usefully thought of in their expanded representation, e.g. RBF or data-defined kernels [Henderson and Titov 05]
- Kernels let us compute with these features implicitly
  - Example: implicit dot product in quadratic kernel takes much less space and time per dot product
  - Of course, there’s the cost for using the pure dual algorithms…